Handout: Plasticity and Hardening

Part I: Stress-Strain Curve

The stress-strain curve or diagram gives a direct indication of the material properties. Hooke's Law (Robert Hooke, 1678) $\sigma = E\varepsilon$ describes the elastic behaviour (up to yielding point) of materials for a bar subjected to uniaxial extension [1].

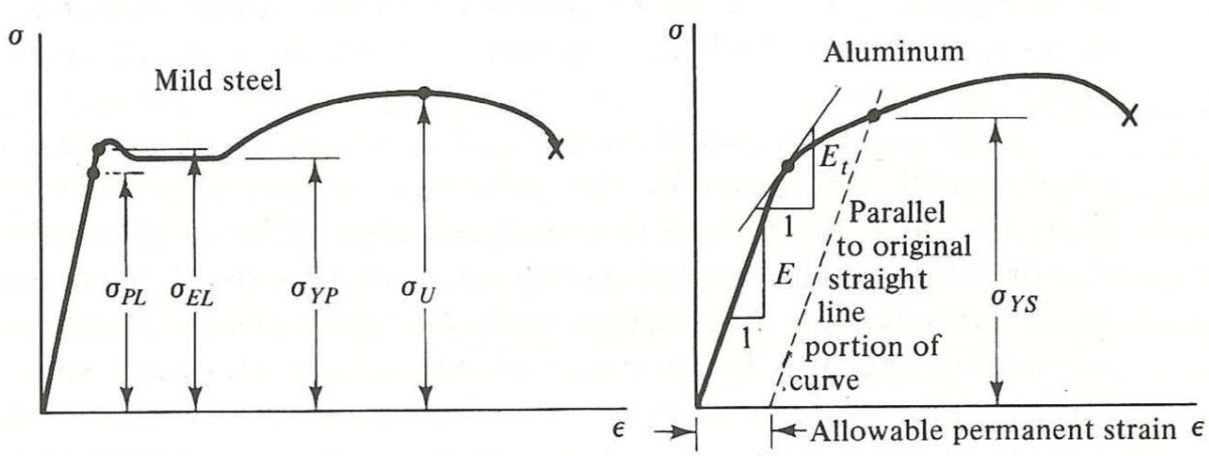

Fig. 1: The meaning of stress-strain curve at different positions [1].

σ_{PL} **Proportional Limit** – Stress above which stress is not longer proportional to strain. Hook's law does not hold anymore.

σ_{EL} **Elastic Limit** – The maximum stress that can be applied without resulting in permanent deformation when unloaded.

σ_{YP} **Yield Point** – Stress at which there are large increases in strain with little or no increase in stress. Among common structural materials, only steel exhibits this type of response.

σ_{YS} **Yield Strength** – The maximum stress that can be applied without exceeding a specified value of permanent strain (typically 0.2%).

σ_U **Ultimate Strength** – The maximum stress the material can withstand (based on the original area)

E **Modulus of Elasticity** – Slope of the initial linear portion of the stress-strain diagram. The modulus of elasticity may also be characterized as the "stiffness" or ability of a material to resist deformation within the linear range

E_t **Tangent Modulus** – Slope of the stress-strain curve above the proportional limit. There is no single value for the tangent modulus; it varies with strain and describes the hardening of the material.

ν **Poisson's ratio** – The ratio of lateral or transverse strain to the longitudinal strain of an uniaxial load case. Poisson's ratios ranges from 0.0 to 0.49

Part II: Non-linear Behaviour

The following Ramberg-Osgood equation is the usual representation of non-linear behavior.

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{\sigma_{YS}}\right)^{n},$$

where ε is the strain, σ is the stress, σ_{YS} is the yield strength, usually the 0.2% proof stress, E is the initial elastic modulus and n is the Ramberg-Osgood parameter (strain hardening exponent) which is a measure of the non-linearity of the curve [2]. Commonly used values of n are about 5 or greater.

The first term of the equation represents the linear behaviour and the second represents the non-linear behaviour, the plastic part. For low stress values, the non-linear component is not significant when compared to the linear component [2].

Tangent Modulus E_t – The tangent modulus is defined as the slope of the stress-strain curve at each value of stress. It is defined by Ramberg-Osgood equation as the inverse of the first derivative in respect to strain [2].

$$E_t = \frac{E\sigma_{YS}}{\sigma_{YS} + 0.002nE\left(\frac{\sigma}{\sigma_{YS}}\right)^{n-1}}.$$

For an idealized elastic-plastic material law the tangent modulus is used to define the slope of the stress-strain curve beyond the yielding point. Actually, the term is confusing since this slope is not indicating an elastic behaviour, not stiffness, but the hardening effect.

Part III: Ductile and Brittle Materials

Ductile Material – Materials that are capable of undergoing large strains (*at normal temperature*) before failure. An advantage of ductile materials is that visible distortions may occur if the loads before too large. Ductile materials are also capable of absorbing large amounts of energy prior to failure. Ductile materials include mild steel, aluminium and some of its alloys, copper, magnesium, nickel, brass, bronze and many others. They fail under shear stresses by yielding.

Brittle Material – Materials that exhibit very little inelastic deformation. In other words, materials that fail in tension at relatively low values of strain are considered brittle. Brittle materials include concrete, stone, cast iron, glass and plaster [1]. They fail under normal tensile stresses. Here we are concentrating on ductile materials.

Part IV: Strain Hardening

Strain hardening, also called work-hardening or cold-working, is the process of making a metal harder and stronger through plastic deformation. When a metal is plastically deformed, dislocations move and additional dislocations are generated. The more dislocations within a material, the more they will interact and become pinned or tangled. This will result in a decrease in the mobility of the dislocations and a strengthening of the material. This type of strengthening is commonly called cold-working. It is called cold-working because the plastic deformation must occur at a temperature low enough that atoms cannot rearrange themselves. When a metal is worked at higher temperatures (hot-working) the dislocations can rearrange and little strengthening is achieved [3].

Strain hardening can be easily demonstrated with piece of wire or a paper clip. Bend a straight section back and forth several times. Notice that it is more difficult to bend the metal at the same place. In the strain hardened area dislocations have formed and become tangled, increasing the strength of the material. Continued bending will eventually cause the wire to break at the bend due to fatigue cracking. After a large number of bending cycles, dislocations form structures called Persistent Slip Bands (PSB). PSBs are basically tiny areas where the dislocations have piled up and moved the material surface out leave steps in the surface that act as stress risers or crack initiation points [3].

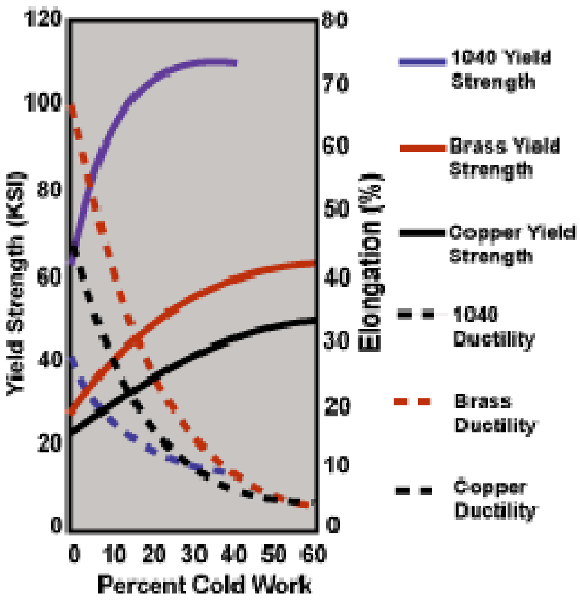

Fig. 2: The effect of strain hardening on yield strength and ductility [3].

It should be understood, however, that increasing the strength by cold-working will also result in a reduction in ductility. The graph shows the yield strength and the percent elongation as a function of percent cold-work for a few example materials. Notice that for each material, a small amount of cold-working results in a significant reduction in ductility [3].

References

[1] http://www.optics.arizona.edu/optomech/references/OPTI_222/OPTI_222_W4.pdf

[2] S. Schneider, S. G. Schneider, H. Marques da Silva, C. de Moura Neto: Study of the non-linear stress-strain behavior in Ti-Nb-Zr alloys; Materials Research. vol.8 no.4, Oct./Dec. 2005

[3] http://www.ndt-ed.org/EducationResources/CommunityCollege/Materials/Structure/strengthening.htm

[4] http://en.wikipedia.org/wiki/Strain_hardening_exponent