Statik

Löse $Ku - F = 0$

Dynamik

$\ddot{u}$ „groß" $\Longrightarrow M\ddot{u} > 0$

Masse

Löse $M\ddot{u} + C\dot{u} + Ku = F$

Zeitintegration

$$\ddot{u} = M^{-1}(F - C\dot{u} - Ku)$$

$\Longrightarrow$ zweimal über t integrieren

$$\int_t^{t+\Delta t} \ddot{u}(\tau)\, d\tau = \dot{u}(t+\Delta t) - \dot{u}(t)$$

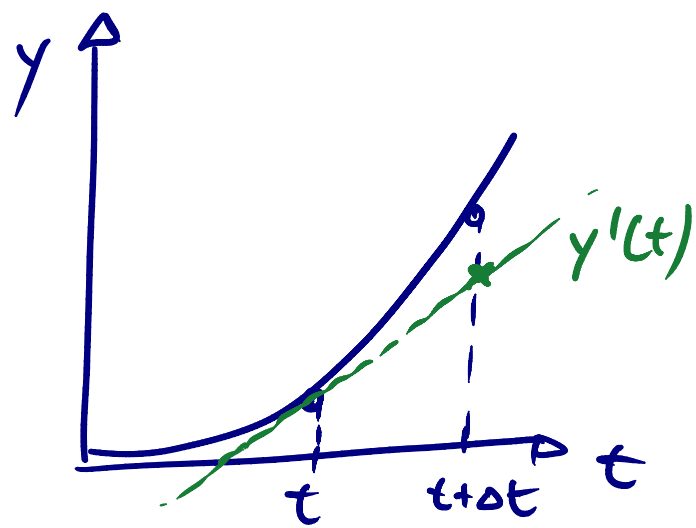
$$\Longleftrightarrow \quad \dot{u}(t+\Delta t) = \dot{u}(t) + \int_t^{t+\Delta t} \ddot{u}(\tau)\, d\tau$$

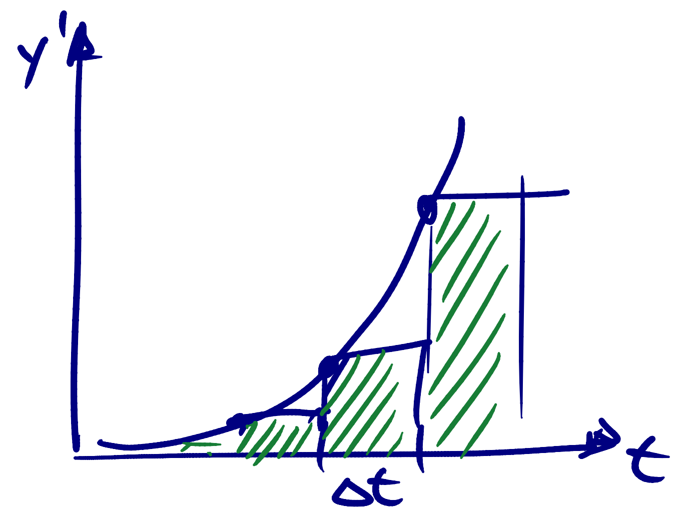
Explizit

sei $y := u$

z.B. expl. Euler:

$$y(t+\Delta t) = y(t) + \underbrace{\Delta t\, y'(t)}_{\approx \int_t^{t+\Delta t} y' \, d\tau}$$

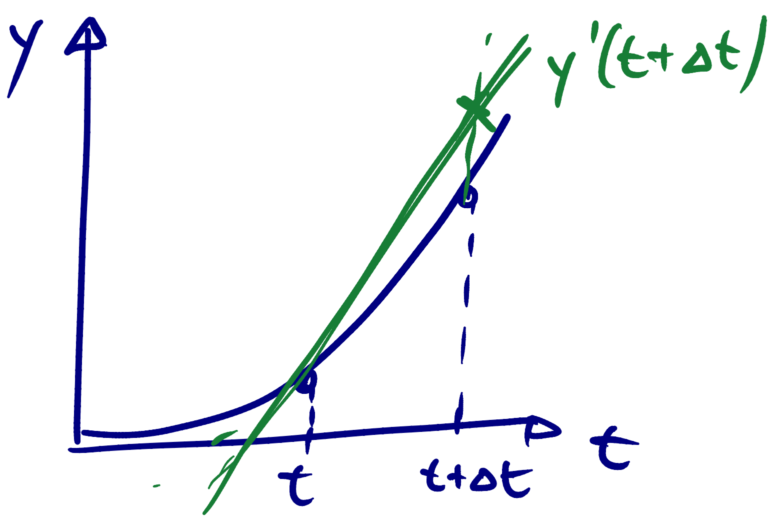


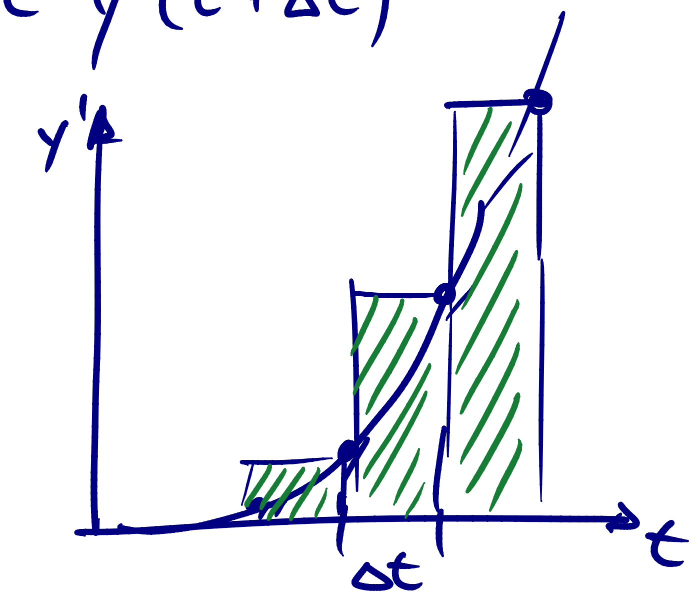


Implizit

z.B. impl. Euler:

$$y(t+\Delta t) = y(t) + \underbrace{\Delta t \; y'(t+\Delta t)}_{\approx \int_t^{t+\Delta t} y' \, d\tau}$$





ANSYS Transient (Theory Reference)

$$\mathbf{M}\ddot{\mathbf{u}}_{i+1} + \mathbf{C}\dot{\mathbf{u}}_{i+1} + \mathbf{K}\mathbf{u}_{i+1} = \mathbf{F}_{i+1}$$

$$\dot{\mathbf{u}}_{i+1} = \dot{\mathbf{u}}_i + \frac{1}{2}\Delta t(\ddot{\mathbf{u}}_i + \ddot{\mathbf{u}}_{i+1})$$

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t\ \dot{\mathbf{u}}_i + \frac{1}{4}\Delta t^2(\ddot{\mathbf{u}}_i + \ddot{\mathbf{u}}_{i+1})$$

$u_i \coloneqq u(t_0 + i\Delta t)$

Abaqus/Explicit (Theory Manual §2.4.5)

$$\dot{\mathbf{u}}_{i+\frac{1}{2}} = \dot{\mathbf{u}}_{i-\frac{1}{2}} + \frac{1}{2}(\Delta t_i + \Delta t_{i+1})\,\ddot{\mathbf{u}}_i$$

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t_{i+1}\,\dot{\mathbf{u}}_{i+\frac{1}{2}}$$

mit

$$\ddot{\mathbf{u}}_i = \mathbf{M}^{-1}(\mathbf{F}_i - \mathbf{C}\dot{\mathbf{u}}_i - \mathbf{K}\mathbf{u}_i)$$

unter der Annahme, daß

$$\dot{\mathbf{u}}_{-\frac{1}{2}} = \dot{\mathbf{u}}_0 - \frac{1}{2}\Delta t_0\,\ddot{\mathbf{u}}_0$$

Terminologie

	ANSYS	Abaqus
Lastschritt	Load step	Step
Zeitschritt	Substep	Increment
Newton-Iteration	Equilibrium iteration	Iteration

Implizit: Große Zeitschritte, löse jeweils (iterativ) ein NLGLS
$(\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} - \mathbf{F} < \mathbf{F}_{\mathrm{crit}})$

Explizit: Viele kleine Zeitschritte, dafür keine Newton-Iterationen notwendig