Hauptseminar Quantenmechanisches Tunneln

Tunnelzeit

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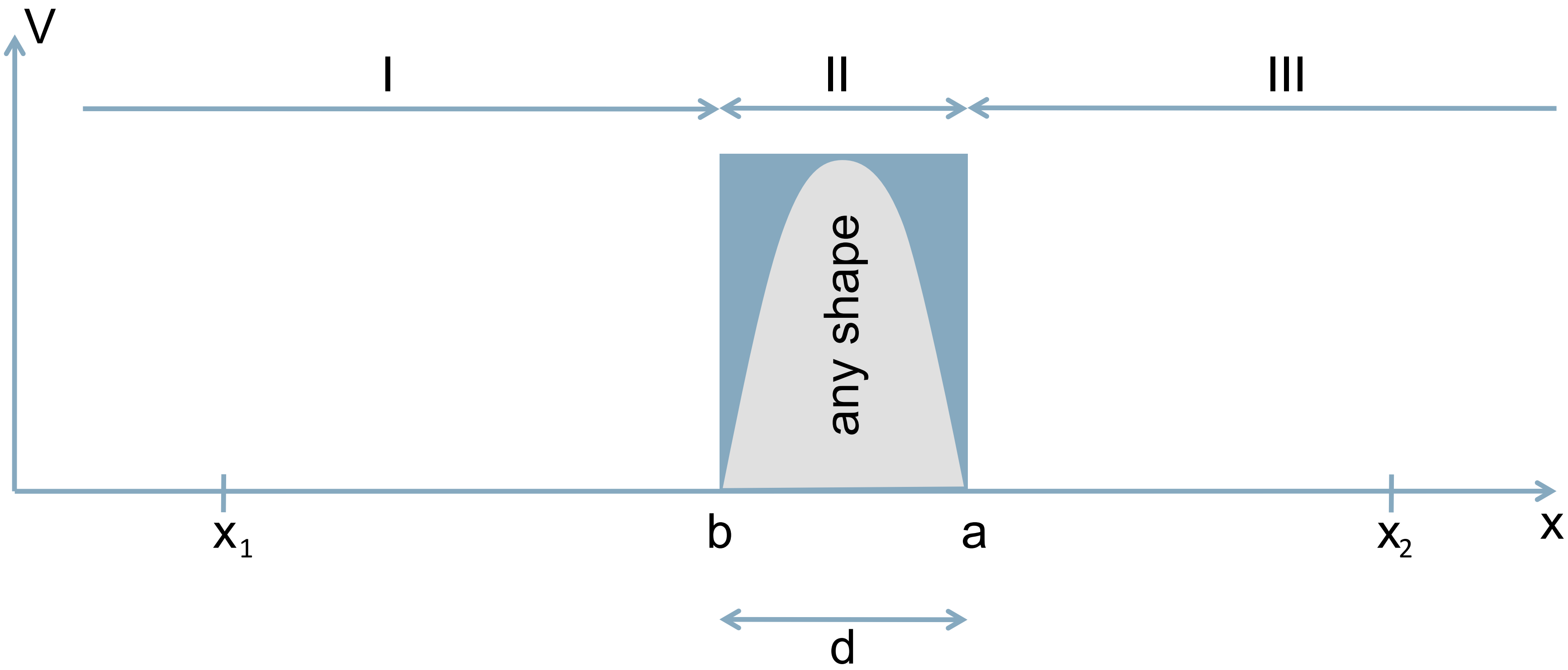
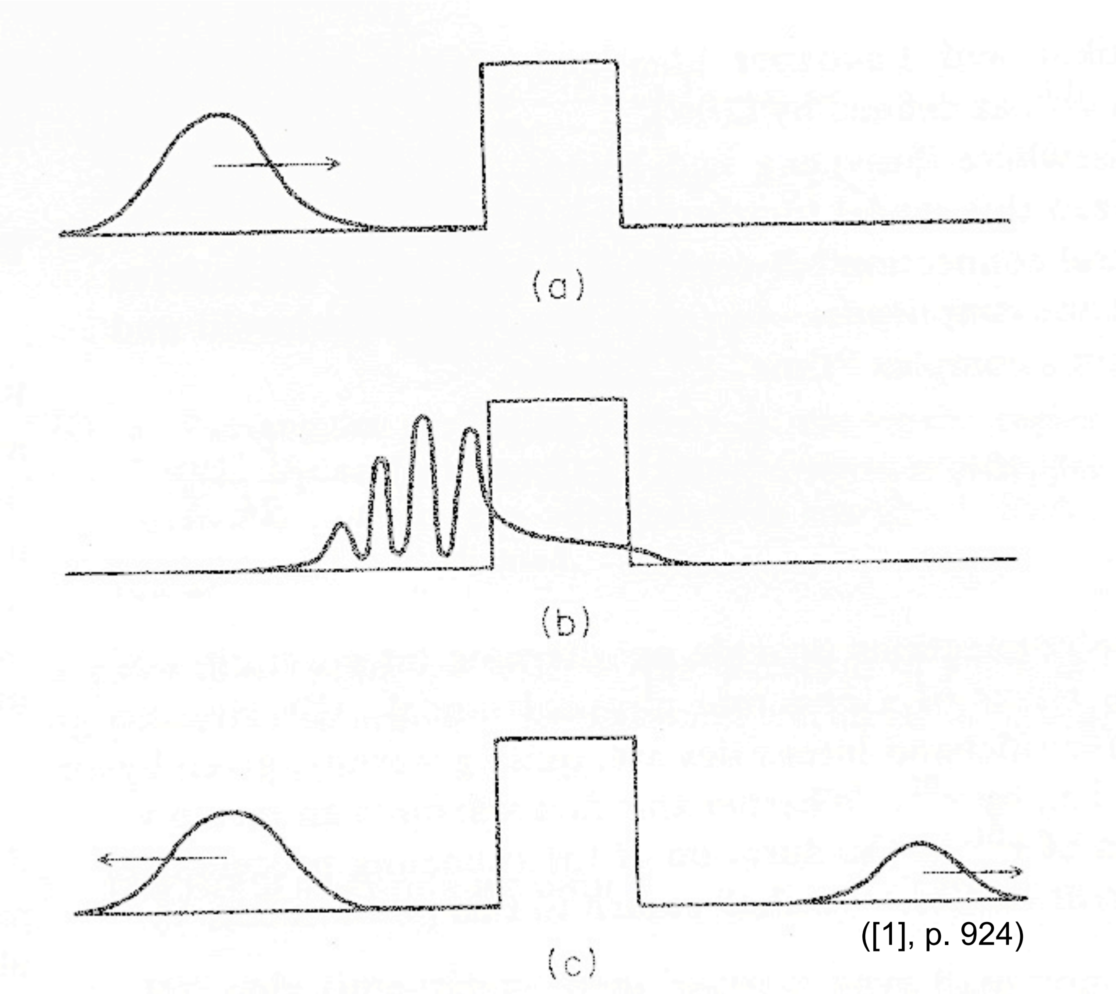
Agenda

Definition of the environment

$$\psi(x) = \begin{cases} \psi_I(x,k) = e^{ikx} + \sqrt{R} e^{i\beta} e^{-ikx} & x < b \\ \psi_{II}(x,k) & b < x < a \\ \psi_{III}(x,k) = \sqrt{T} e^{i\alpha} e^{ikx} & x > a \end{cases} \tag{1}$$



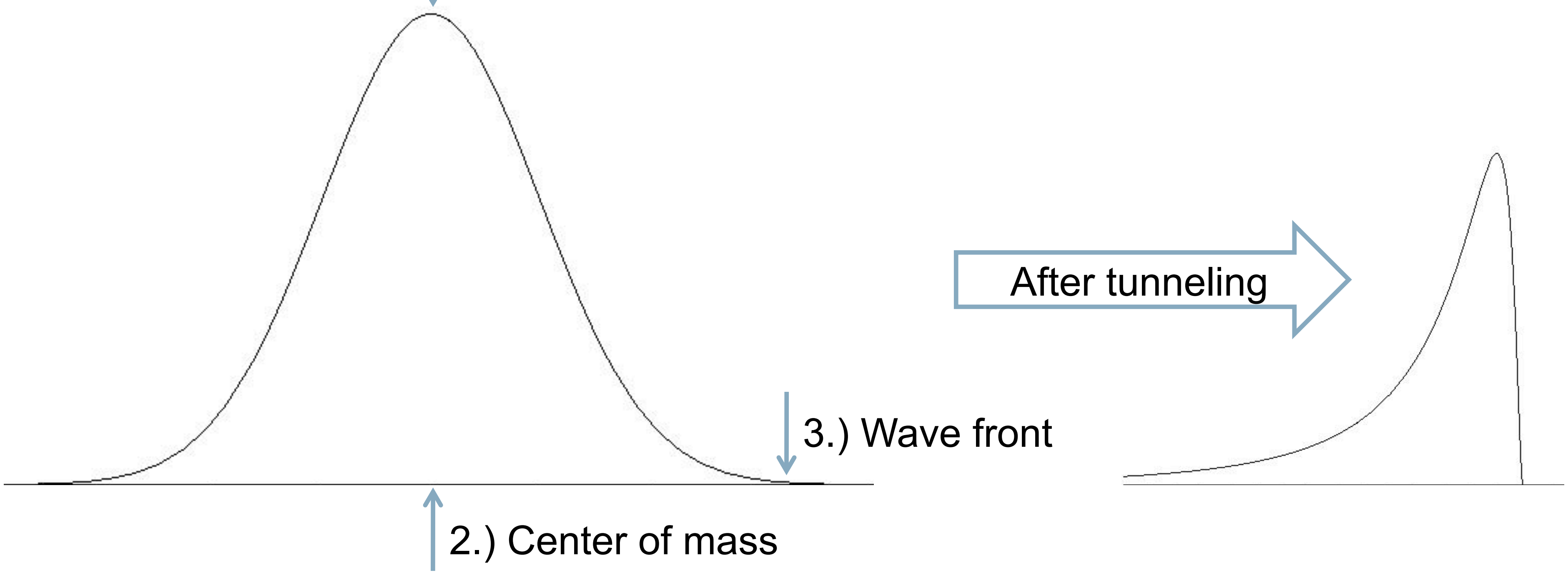
Scheme of reflection and transmission



([1], p. 924)

Three ways of time measurement

1.) The peak could change his position.
2.) The center of mass could change his position.
3.) Incabability of measurement unless you have a rectangular wave-packet.



Definition of different tunneling times

- Dwell time: $\tau_D(x_1, x_2, k)$
- Phase time: $\tau_T^{\varphi}(x_1, x_2, k)$
- Extrapolated phase time: $\Delta\tau_T^{\varphi}$
- Local Larmor time: τ_{yT}^{L}
- Büttiker-Landauer time: τ_T^{BL}

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Characteristics

$$\tau_D(x_1, x_2, k) = \frac{1}{v(k)} \int_{x_1}^{x_2} dx \mid \psi(x,k) \mid^2 \quad \underset{x_1=b, x_2=a}{\overset{d \gg 1}{\approx}} \quad \frac{2mk}{\hbar \kappa k_0^2} \tag{2}$$

$$\Longrightarrow \lim_{k \to 0} \tau_D(d, k) = 0 \tag{3}$$

Problems of the dwell time:

- v(k) is in general not constant
- The dwell time is an average time taken over both channels
- Mathematical approach → Could not be checked in an experiment

Usage: Revise other results

If you assume, that the dwell time is a good expression for the tunneling time, you can compare other results for the tunneling time to the dwell time. Especially in two cases it is quite easy:

- $d = 0 \Longrightarrow R = 0 \Longrightarrow \tau_D(x_1, x_2, k) = \frac{x_1 - x_2}{v(k)}$
- $d \to \infty \Longrightarrow T = 0 \Longrightarrow \tau_D(x_1, x_2, k) = 0$

For any other case, consider the convex combination of your result:

- $\tau_D(x_1, x_2, k) = R \cdot \tau_R + T \cdot \tau_T$

The results for this comparison are integrated in the following presentation of the other times.

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Definitions for a rectangular potential

$$k_0^2 = \frac{2mV_0}{\hbar}, \quad k^2 = \frac{2mE}{\hbar}, \quad \kappa^2 = k_0^2 - k^2 \tag{4}$$

$$b = 0 \tag{5}$$

$$e^{ikx+i\omega t} \overset{\texttt{Barrier}}{\Longrightarrow} \begin{cases} \text{Reflection:} & \sqrt{R}e^{i\beta(k)}e^{ikx+i\omega t} \\ \text{Transmission:} & \sqrt{T}e^{i\alpha(k)}e^{ikx+i\omega t} \end{cases} \tag{6}$$

Derivation of the phase time

$$\sqrt{T(k)}\exp\left(i\alpha(k) + ikx - \frac{iE(k)t}{\hbar}\right) \tag{7}$$

$$\overset{\frac{d}{i\cdot dk}(7)}{\Longrightarrow} \frac{d}{dk}\alpha(k) + x_{peak} - \frac{dE(k)}{dk}\frac{t}{\hbar} \overset{!}{=} 0 \tag{8}$$

$$\Longrightarrow v(k) = \frac{\hbar k}{m} = \text{const.}, \quad \delta_T = \frac{\alpha'}{v(k)} \tag{9}$$

Derivation of the phase time

The phase time is defined by v(k) and the distance between the two points at each side of the barrier ($x_{1/2}$) plus a third term representing the delay caused by the barrier.

$$\tau_T^{\varphi}(x_1, x_2, k) = \frac{1}{v(k)} \cdot (x_2 - x_1 + \alpha'(k)) \tag{10}$$

Searching for an expression for the time in the barrier, you can build the limes for x_1 and x_2 to b and a, once again assuming a constant velocity. This is called the extrapolated phase time:

$$\Delta\tau_T^{\varphi}(b, a, k) = \frac{1}{v(k)} \cdot (a - b + \alpha'(k)) = \frac{1}{v(k)} \cdot (a - \alpha'(k)) \tag{11}$$

Derivation of the phase time

According to the proceedings with the times for the transmission, the phase time and the extrapolated phase time for the reflection can be defined:

$$\tau_R^{\varphi}(x_1, k) = \frac{1}{v(k)} \cdot (-2x_1 + \beta'(k)) \tag{12}$$

$$\Delta\tau_R^{\varphi}(b, k) = \frac{1}{v(k)} \cdot (-2b + \beta'(k)) = \frac{\beta'(k)}{v(k)} \tag{13}$$

Assuming a high potential with a not infinitesimal small width and a particle with a low energy, the extrapolated phase times can be approximated with:

$$\kappa d \gg 1 \Longrightarrow \Delta\tau_R^{\varphi}(d, k) \simeq \frac{2m}{\hbar k \kappa} \simeq \Delta\tau_T^{\varphi}(d, k) \tag{14}$$

Problems of the phase time

Check with the dwell time:

$$\lim_{k \to 0} \tau_D(x_1, x_2, k) \overset{(3)}{=} 0 \neq \lim_{k \to 0} \frac{2m}{\hbar \kappa k} = \infty \qquad (15)$$

- Interference at the barrier is not considered
- v(k) is in general not constant
- measurement in relation to the peak of the wave package

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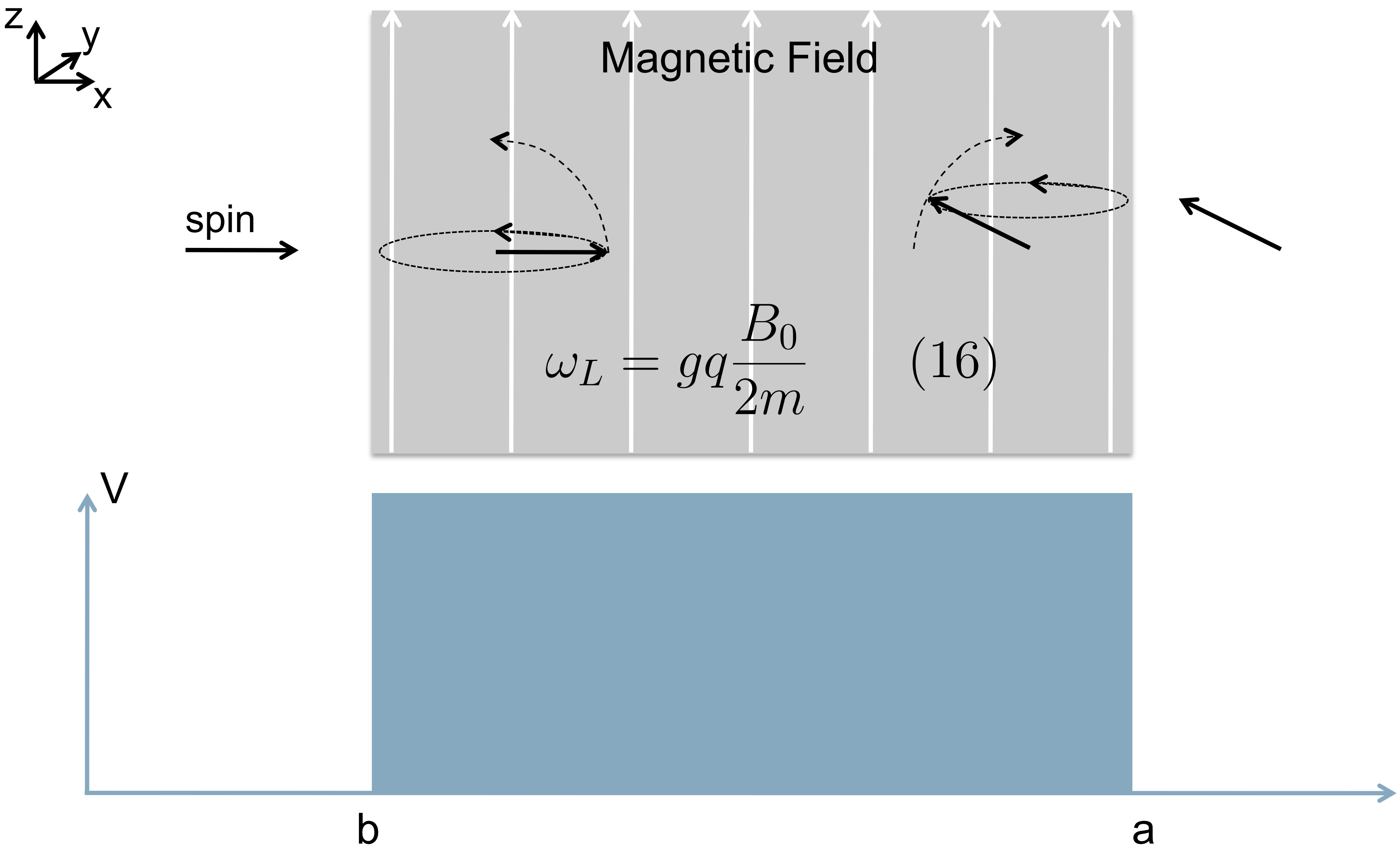
Semiclassical background of Larmor time



$$\omega_L = gq\frac{B_0}{2m} \qquad (16)$$

Definition of the Larmor times

For the local Larmor times both components are used to define tunneling times. Therefore you always consider the average of the spins. First Rybachenko took the y-component to define the first Larmor time:

$$\tau_{yT}^{L}(x_1, x_2, k) = \lim_{\omega_L \to 0} \frac{\langle s_y \rangle_T}{-\frac{1}{2}\hbar\omega_L} \tag{17}$$

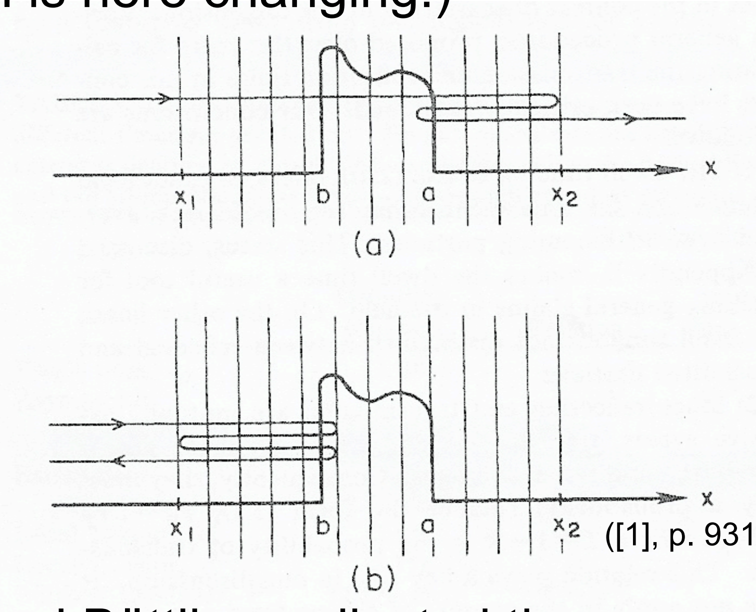
He came to the result, that this time is equal to the dwell time.

In the same approach, Büttiker defined the Larmor time for the z-component of the spin. He had the opinion, that under special circumstances the transmitted particles prefer the spin up component.

$$\tau_{zT}^{L}(x_1, x_2, k) = \lim_{\omega_L \to 0} \frac{\langle s_z \rangle_T}{-\frac{1}{2}\hbar\omega_L} \simeq \frac{md}{\hbar\kappa} \tag{18}$$

Problem of this definitions

• Impossibility to create a magnetic field with a sharp border
• Reflections at the beginning and the end of the magnetic field are possible (the potential is here changing!)



([1], p. 931)

• Both, Rybachenko and Büttiker, adjusted the x-component in the positive direction in front of the potential. Afterwards, they cannot measure the y- or z-component exactly, because the commutator is not zero.

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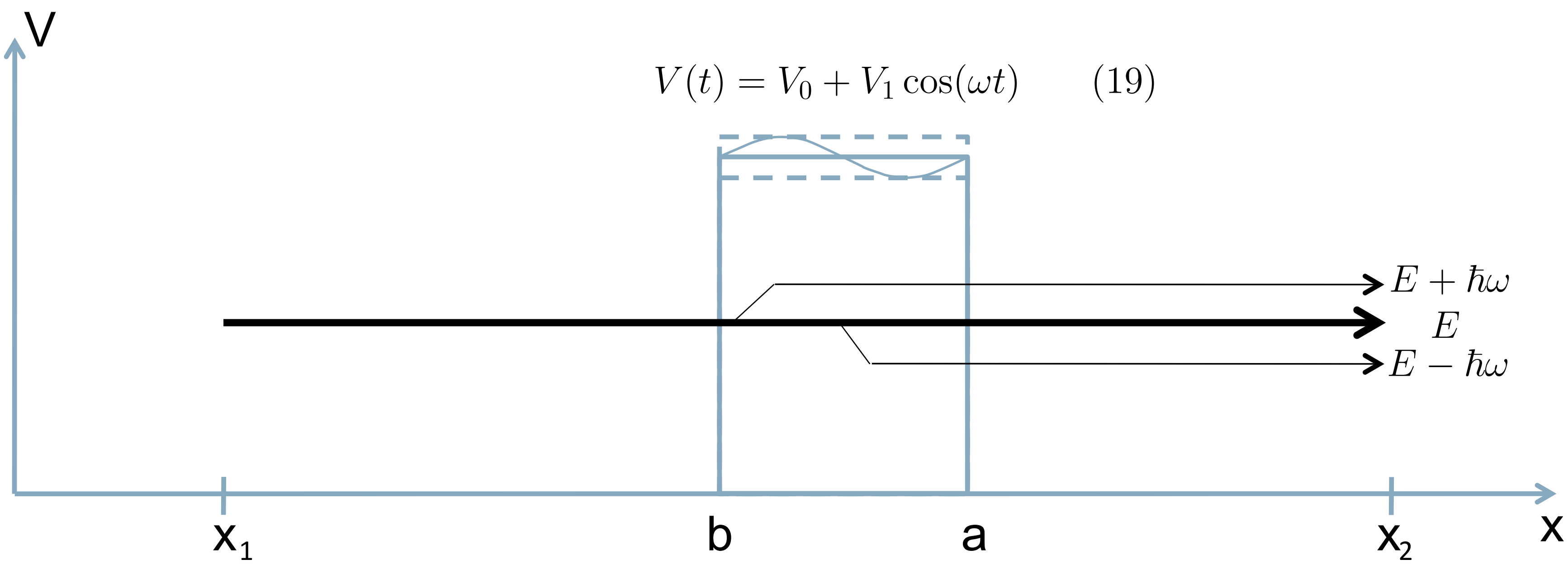
The approach of Büttiker-Landauer

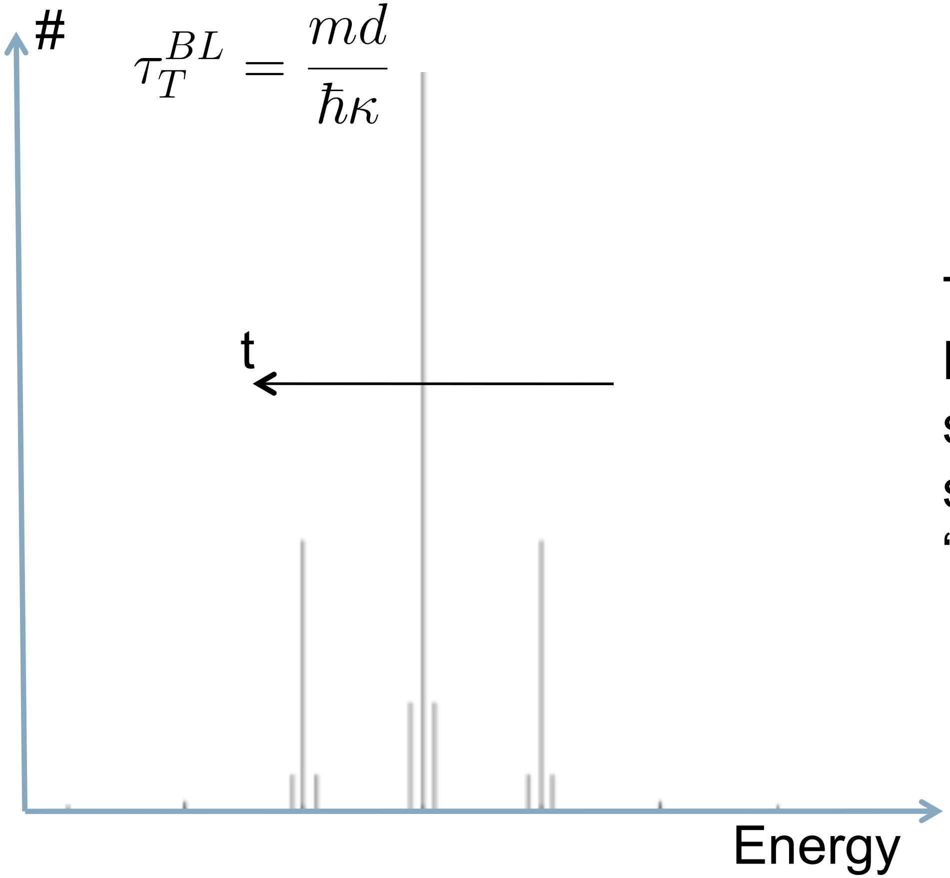


Particles with a higher or lower energy are measured for a wide barrier and a high frequency.
They can absorb energy while the potential is increasing or emit energy while the potential is decreasing.

The approach of Büttiker-Landauer

$$I^T_{\pm 1}(\omega) \equiv \left| \frac{A_{\pm 1}(\omega)}{A_0} \right|^2 = \left(\frac{V_1}{2\hbar\omega} \right)^2 \left(\exp\left(\pm\omega \frac{md}{\hbar\kappa} \right) - 1 \right)^2 \quad (20)$$

$$\tau^{BL}_T = \frac{md}{\hbar\kappa} \quad (21)$$



This expression is equal to the Büttiker's Larmor time. But it should be discussed, why this should be the tunneling time in this "experiment".

Are tunneling times real?

So far, all considered tunneling times are real. If you assume, that the wave inside the barriere is still a plain wave (with a complex exponent), you can sketch a reason for a complex tunneling time.

$$\psi_{II} = \exp\left(i \cdot x\sqrt{\frac{2m}{\hbar^2}(E - V_0)}\right) = \exp\left((i \cdot x)\underbrace{\left(i\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}\right)}_{=:\gamma}\right) \tag{22}$$

$$\gamma = \frac{i \cdot p}{\hbar} = \frac{i \cdot m \cdot v}{\hbar} = \frac{i \cdot m \cdot s}{\hbar \cdot t} \overset{!}{\in} \mathbb{R} \tag{23}$$

$$m, s, \hbar \in \mathbb{R} \Longrightarrow t = i\tau, \tau \in \mathbb{R} \tag{24}$$

In example, Sokolovski and Baskin proposed such a complex time defined over a path integral.

Agenda

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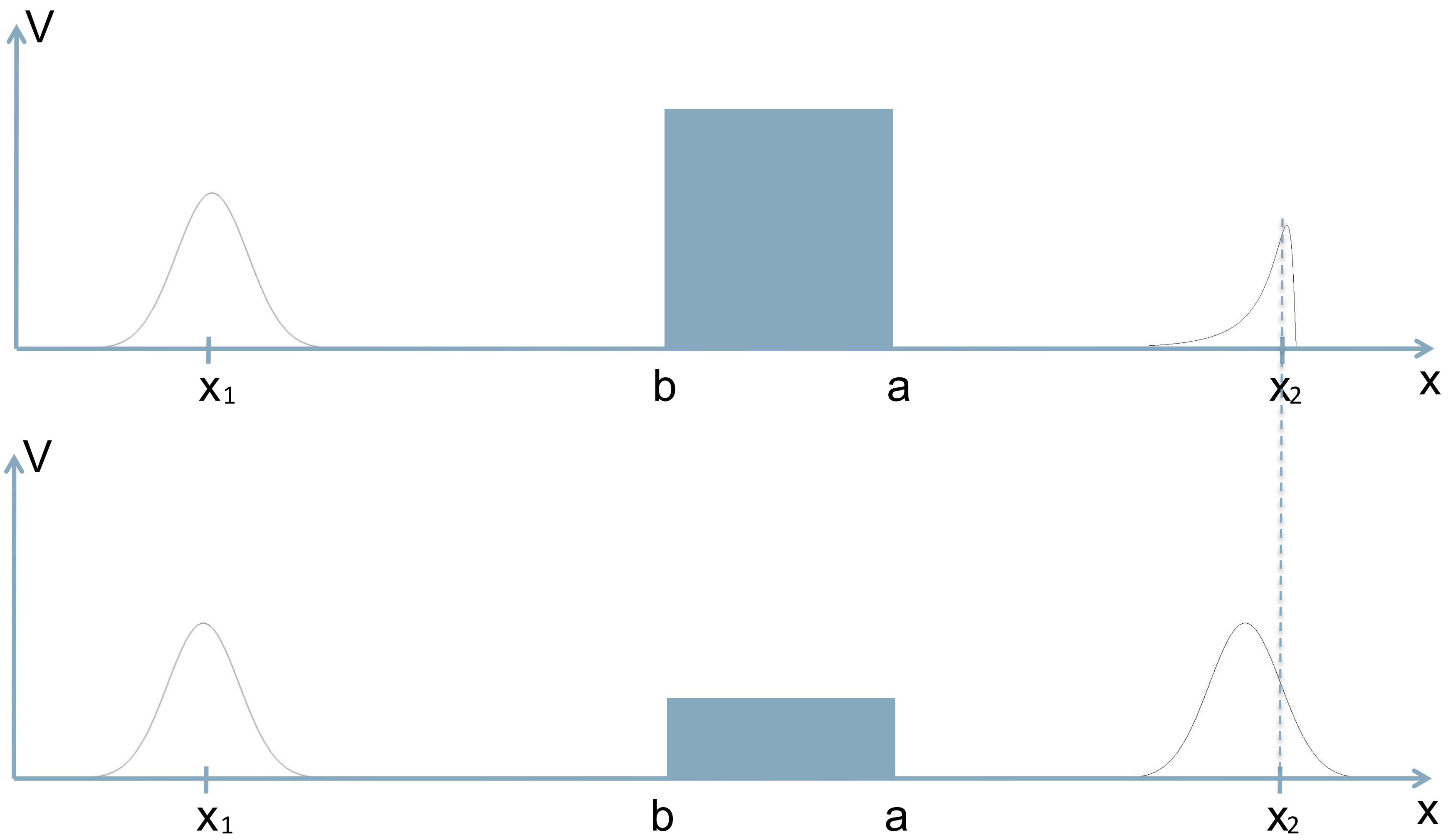
Nimtz – Scheme of his experiment



Nimtz – A critical view

• He used microwaves for the both potentials.

• The first potential is higher than the energy of the incoming wave. For that reason the wave packet has to tunnel through the barrier.

• In contrast the second potential is below the energy of the wave. If you assume a classical case, the barrier does not matter.

• The information of the wave in the first case arrives earlier at the detector. Therefore Nimtz suggested, that the tunneling waves are faster.

<u>Criticism:</u>

•Taking a classical perspective to explain a problem in quantum mechanics

• Measurement of the wave in relation to the peak (peak is not at a constant k for the tunneling wave)

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Conclusion

• As the mathematical and the experimental calculated real times seem to be false, the definition of an imaginary tunneling time is almost the only possibility left.

• Based on the different approaches, it is clear, that each concept is trying to avoid the absence of a time-operator in quantum mechanics by defining an expression with the unity of a time.

• The main problem of all concepts is, that these theories are based either on classical quantities (i.e. the definition of a time as the quotient of a distance over a velocity) or on a classical concept.

Questions?

Thank you for listening.

Bibliography:

• [1]: Reviews of Modern Physics, Vol. 61, No. 4, 1989: E.H. Hauge and J.A. Støvneng, "Tunneling times: a critical review"
• [2]: http://www.ethlife.ethz.ch/archive_articles/081205_keller_tunneling_sch/index
• [3]: http://theory.gsi.de/~vanhees/faq/nimtz/node7.html

Appendix

- Dwell time: The definition of the probability in (25) is well-known. The dwell time is defined by the integration of this probabilty over dt.

$$P(t, x_1, x_2) = \int_{x_1}^{x_2} dx \mid \psi(x,t) \mid^2 \tag{25}$$

$$\bar{\tau}_D(x_1, x_2) = \int_0^\infty dt P(t, x_1, x_2) \tag{26}$$

- Unity check for the approximation of the dwell time in (2):

$$\frac{[m]\cdot[k]}{[\hbar]\cdot[\kappa]\cdot[k_0]^2} = \frac{[m]}{[\hbar]\cdot[k_0]^2} = \frac{kg}{Js\cdot\frac{1}{m^2}} = \frac{kg\cdot m^2}{\frac{kg\cdot m^2}{s^2}s} = s \tag{27}$$

- The relation between Büttiker‘s Larmor time and the Büttiker-Landauer time:

$$\langle s_z\rangle_T \gg \langle s_y\rangle_T \Longrightarrow \tau_T^{BL} = \sqrt{\left(\tau_{yT}^L\right)^2 + \left(\tau_{zT}^L\right)^2} \approx \tau_{zT}^L \tag{28}$$

Appendix

Keller's way to measure tunneling times

• Some theoretical physics are of the opinion that an electron needs only between 500 and 600 attoseconds for tunneling through a laser barriere. There was no way to measure such a short time until now.

• Prof. Dr. Ursula Keller and her team are the first who can measure such a short time with a femtosecond laser puls. This puls is circular polarised and it needs for a 360 ° turn only 2,4 femtoseconds. For that reason it is possible to measure time with a fault of twelve attoseconds.

• But also with this circular polarised laser it was not possible to measure the tunneling time of a particle. The explanation of this result by Prof. Dr. Keller was the idea that the tunneling time is not a time in classical case.