1$^{\text{st}}$ Ultimate League for Mastery in Applied Economics (ULMA)

Friday, 6$^{\text{th}}$ December 2024   Duration: 120 Minutes

Institute of Economics   Luca Cermak, M.Sc. & Simon Maier, M.Sc.

To be completed by the candidate:

Last name: _____ First name: _____ Matriculation number:

Course name: _____ Target degree: _____ _____

_____
Date, Signature

I hereby declare that I am eligible to take the test.

Notes on the test:

- All tasks are to be processed.

- Check that you have received all 5 pages.

- Only use the space provided to you under the task.
  The back page should only be used for corrections.

- If an intermediate result is given, continue with this!

- Give reasons for your answers! Document your calculation method for calculations!

- Round to 2 decimal places.

- You can answer in either German or English.

Permitted aids:

- document-proof pens, set square or ruler, non-programmable pocket calculator, foreign language dictionary.

To be completed by the examiner:

Task	1	2	3	4	5	6	7	8	9	10	$\sum$
Possible points	10	10	10	10	10	10	10	10	10	10	**100**
Achieved points											

Grade: _____

Test Question (10 P.)

Alma has the choice: she can either have 24 hours of leisure F or work in a bakery to bake bread B. Her utility can be represented by the following utility function:

$$U(F, B) = \frac{2}{3}\sqrt{F} + \frac{1}{3}B$$

For the production or baking of bread, the following applies:

$$B(F) = 2\sqrt{24 - 2F}$$

The bakery belongs to Kurt, who does not bake bread himself. Kurt values bread as follows:

$$U_K(B_K) = \ln(1 + B_K)$$

Both parties can generate and share a joint surplus. If Alma does not bake bread, she receives basic provision from the state amounting to $B_R = \frac{1}{10}$ loaves of bread. Kurt initially does not charge Alma rent for the bakery.

a) Show that the marginal rate of transformation is given by
$$\text{MRT} = \frac{dB(F)}{dF} = -\frac{2}{\sqrt{24 - 2F}}$$

<div align="right">(1 P.)</div>

b) Explicitly derive Alma's indifference curve (for utility level $\overline{U}$) and show that the marginal rate of substitution is given by

$$\text{MRS} = \frac{dB(F)}{dF} = -\frac{1}{\sqrt{F}}$$

(1 P.)

c) Show that Alma optimally chooses $F^* = 4$ hours of leisure and spends the rest of her time baking $B^* = 8$ loaves of bread. (2 P.)

Hint: Use the results from a) and b).

d) Show that Alma's utility at the optimum is $U^* = 4$. Additionally, demonstrate that it is beneficial for Alma to work in the bakery. (2 P.)

e) Assume Alma wants to start horseback riding. Since taking care of a horse requires more time, she would need to allocate $F_{\text{horse}} = 10$ hours of leisure.

How many loaves of bread would she now bake? What would her new utility U_{total} be in this case if horseback riding gave her an additional additive utility of $U_{\text{horse}} = \frac{5}{3}$, so that her utility is given by $U(F_{\text{horse}}, B_{\text{horse}}) + U_{\text{horse}}$? Would she start horseback riding? (2 P.)

f) Which situation would Kurt prefer if Alma had to pay 10% of her loaves of bread as rent? Which situation would he prefer if his utility function

$$U_K(B_K) = -B_K$$

would be? Give a brief explanation. (2 P.)