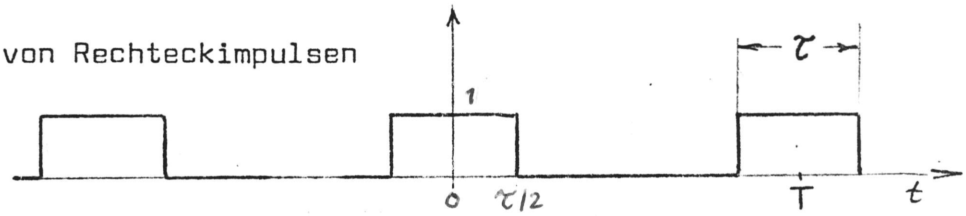
Beispiele:

1) Periodische Folge von Rechteckimpulsen



$f(t) = f(t + T)$

$f(t) = f(-t)$

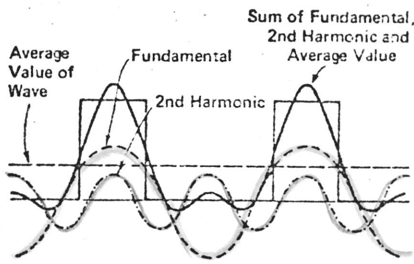
$f(t) = 1 \qquad 0 \le t < \frac{\tau}{2}\ ;\ T - \frac{\tau}{2} < t \le T \qquad \left(f(t) = \frac{1}{2} \text{ an den Sprungstellen}\right)$

$f(t) = 0 \qquad \frac{\tau}{2} < t < T - \frac{\tau}{2}\ ; \quad f(t) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos n\omega t; \quad \omega = \frac{2\pi}{T}$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

$$a_n = \frac{4}{T}\int_0^{\tau/2} \cos n\omega t\, dt = \frac{4}{n\omega T}\ \sin n\omega t \Big|_0^{\tau/2}$$

$$a_n = 2\,\frac{\tau}{T}\cdot\frac{\sin(n\omega\tau/2)}{n\,\frac{\omega\tau}{2}}\ ; \qquad a_0 = 2\,\frac{\tau}{T}$$



somit:

$$f(t) = \frac{\tau}{T} + 2\,\frac{\tau}{T}\sum_{n=1}^{\infty}\frac{\sin\frac{n\omega\tau}{2}}{\frac{n\omega\tau}{2}}\ \cos n\omega t$$

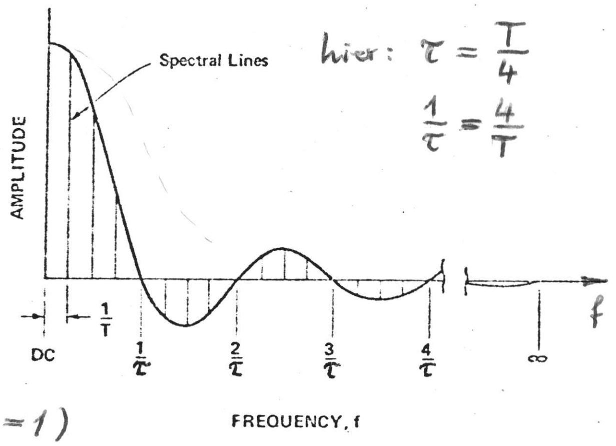
Darstellung der a_n :

$a_n \sim \frac{\sin x}{x}$ mit $x = \frac{n\omega\tau}{2}$

mit $f = \frac{n}{T} \qquad a_n \sim \frac{\sin \pi\tau f}{\pi\tau f}$

Nullstellen: $\pi\tau f = \pi,\ 2\pi,\ 3\pi, \ldots$

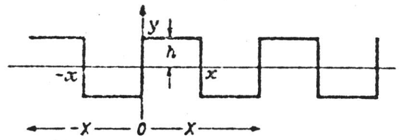
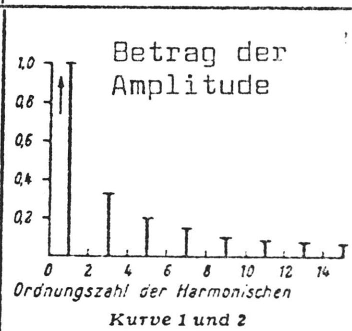
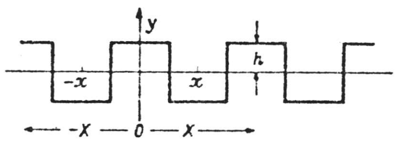
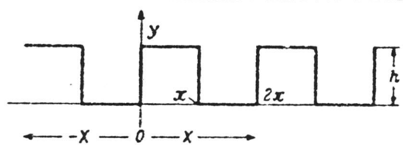
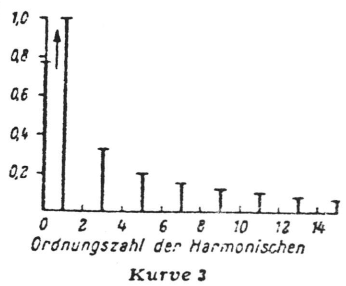
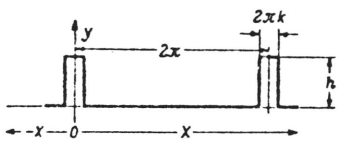
$f = \frac{1}{\tau}, \frac{2}{\tau}, \frac{3}{\tau}, \ldots$



hier: $\tau = \frac{T}{4}$

$\frac{1}{\tau} = \frac{4}{T}$

Spezialfälle (Periode 2π) : $(d.h.\ \omega = 1)$

	Kurvenverlauf	Gleichung	Oberwellenaufbau
1		$f(x) = \frac{4h}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \frac{\sin 9x}{9}\ldots\right)$	 Betrag der Amplitude; Ordnungszahl der Harmonischen; Kurve 1 und 2
2		$f(x) = \frac{4h}{\pi}\left(\cos x - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \frac{\cos 7x}{7} + \frac{\cos 9x}{9}\ldots\right)$	
3		$f(x) = \frac{h}{2} + \frac{2h}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5}\ldots\right)$	 Ordnungszahl der Harmonischen; Kurve 3
4		$f(x) = h\left\{k + \frac{2}{\pi}\left(\sin k\pi \cos x + \frac{1}{2}\sin 2k\pi\cdot\cos 2x + \frac{1}{3}\sin 3k\pi\cos 3x\ldots\right)\right\}$	