A change-point problem for m-dependent multivariate random field

Vitalii Makogin[a,*], Duc Nguyen[a,**]

[a]*Institute of Stochastics, Ulm University,
Helmholtzstraße 18, Ulm, 89075, Baden-Württemberg, Germany*

Abstract

In this paper, we consider the change-point problem over a centered, stationary and m-dependent multivariate random field. Under the distribution free assumption, a change-point test using CUSUM statistic is proposed to detect anomalies within a multidimensional random field, controlling the false positive rate as well as the Family-wise error in the multiple hypotheses testing context.

Keywords:
CUSUM, multiple hypotheses testing, anomaly detection

1. Introduction

The change-point problem is a fundamental issue with broad applications in various fields such as quality control, economics, medicine, and environmental science, to detect any abrupt variations. The theory about the change-point problem was studied thoroughly in [9, 10, 5] as well as its application for different types of data such as time series, images [2, 1, 6], just to name a few. Among the numerous techniques developed for change-point detection, the Cumulative Sum (CUSUM) method, which is a sequential analysis technique traditionally used in quality control for monitoring changes in the mean level of a process, has proven its simplicity, robustness, and effectiveness. In the book [4, 5], change-point problems were studied within a general parametric framework utilizing a CUSUM statistic test, which is widely recognized for anomaly detection, particularly in time series; see, for example, [13, 8].

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[*]Dedicated to the memory of Dr. Vitalii Makogin (12.12.1987 - 08.05.2024)

[**]Corresponding author

Email addresses: `vitalii.makogin@uni-ulm.de` (Vitalii Makogin),
`tran-1.nguyen@uni-ulm.de` (Duc Nguyen)

In this paper, we develop a CUSUM test to detect anomalies within a centered, stationary and m-dependent multivariate random field based on the methodology suggested in [6]. The paper is organized as follows: In Section 2, the model of anomaly detection over a random field is given, where the existence of anomaly is indicated through a hypothesis testing procedure. In Section 3, we obtain an upper bound for the tail probability of the test statistic, allowing one to obtain the global critical value in which the type-I error is controlled. Section 4 shows the numerical results based on the realizations of a m-dependent Gaussian random field.

2. Problem Setting

2.1. Random field with change in mean

For n and d positive integers, let $\left\{\xi_k \in \mathbb{R}^n, k \in \mathbb{Z}^d\right\}$ be a centered, stationary, m-dependent and real-valued random field. For each $\theta \in \Theta$, define the corresponding set $I_\theta \in \mathbb{Z}^d$ as a set of scanning windows. Our goal is to detect anomalies in the space $\mathbb{Z}^d$ depending only on $\theta \in \Theta$. We only consider anomalies in a window $W \subset \mathbb{Z}^d$, henceforth, $I_\theta \subset W$ and let $I_\theta^c = W \setminus I_\theta$. Assume that $W = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_d, b_d]$, where $a_i, b_i \in \mathbb{Z}^+$ for all $i = 1, 2, \ldots, d$. Nevertheless, it is important to have some restrictions on I_θ, we need to consider some significant levels of the set I_θ to assure that whether it is a major part of the window W or there is not an inconsiderable amount of anomalies. In other words, for $\gamma_1 < \gamma_2$ and $\gamma_1, \gamma_2 \in (0, 1)$, from a practical point of view, we choose $\gamma_0 = 0.05$ and $\gamma_1 = 0.5$. Denote by

$$\Theta_0 = \left\{\theta \in \Theta, \gamma_0 |W| \leq |I_\theta| \leq \gamma_1 |W|\right\}.$$

Inversely, the set of parameters $\Theta_1 = \Theta \setminus \Theta_0$ represents for huge or extremely small of anomalies, i.e,

$$\Theta_1 = \left\{\theta \in \Theta, |I_\theta| < \gamma_0 |W|, \text{ or } |I_\theta| > (1 - \gamma_1)|W|\right\}.$$

So far, we have defined the parametrized set of scanning windows imposed by some conditions. The presence of cracks can be studied by testing a null hypothesis, i.e., there is no cracks within the window W. Therefore, it is needed to construct a CUSUM test statistic and propose a rejection rule.

2.2. Hypothesis testing

Assume that there exists at least one anomaly region I_{θ_0} in the window W and we observe values

$$s_k = \xi_k + h \mathbb{1}\left\{k \in I_{\theta_0}\right\},$$

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where h is a fixed vector in $\mathbb{R}^n$ and unknown. The vector h can be considered as the change in mean of I_{θ_0} and its complement $W \setminus I_{\theta_0}$. Hence, we consider the variation of the expectations of the random field $\{\xi_k, k \in \mathbb{Z}^d\}$ by testing the two following hypotheses:

$H_0 : \mathbb{E}\xi_k = \mu$ for all $k \in W$, i.e., there no change in mean, versus

H_1: there exists a vector $h \in \mathbb{R}^n, h \neq \vec{0}$ in mean, i.e. $\mathbb{E}\xi_k = \mu + h, k \in I_\theta$ and $\mathbb{E}\xi_k = \mu, k \in I_\theta^c$

For a fixed window $W \subset \mathbb{Z}^d$, one has a sample $S = \{s_k \in \mathbb{R}^n, k \in \mathbb{Z}^d\}$ and

$$
\begin{aligned}
L(\theta) &= \frac{1}{|I_\theta|} \sum_{k \in I_\theta} s_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} s_k \\
&= \frac{1}{|I_\theta|} \sum_{k \in I_\theta} (\xi_k + h\mathbb{1}\{k \in I_{\theta_0}\}) - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} (\xi_k + h\mathbb{1}\{k \in I_{\theta_0}\}) \quad (1) \\
&= \frac{1}{|I_\theta|} \sum_{k \in I_\theta} \xi_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} \xi_k + h\left(\frac{|I_\theta \cap I_{\theta_0}|}{|I_\theta|} - \frac{|I_\theta^c \cap I_{\theta_0}|}{|I_\theta^c|}\right).
\end{aligned}
$$

For testing purposes, employ the following statistic:

$$
T_W(S) = \max_{\theta \in \Theta_0} \|L(\theta)\|_p = \max_{\theta \in \Theta_0} \left\| \frac{1}{|I_\theta|} \sum_{k \in I_\theta} s_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} s_k \right\|_p. \quad (2)
$$

A rejection rule requires a threshold y_α such that if $T_W(S)$ exceeds y_α, we reject the null hypothesis H_0. However, we need to compute the probability of type I error $\mathbb{P}_{H_0}(\max_{\theta \in \Theta_0} |L(\theta)| \geq y_\alpha)$. It yields,

$$
\begin{aligned}
&\mathbb{P}_{H_0}\left(\max_{\theta \in \Theta_0} |L(\theta)| \geq y_\alpha\right) \\
&= \mathbb{P}_{H_0}\left(\max_{\theta \in \Theta_0} \left| \frac{1}{|I_\theta|} \sum_{k \in I_\theta} \xi_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} \xi_k + h\left(\frac{|I_\theta \bigcup I_{\theta_0}|}{|I_\theta|} - \frac{|I_\theta^c \bigcup I_{\theta_0}|}{|I_\theta^c|}\right) \right| \geq y_\alpha\right) \\
&= \mathbb{P}\left(\max_{\theta \in \Theta_0} \left| \frac{1}{|I_\theta|} \sum_{k \in I_\theta} \xi_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} \xi_k \right| \geq y_\alpha\right).
\end{aligned}
$$

Denote by

$$
\sum_{k \in W} b_k \xi_k := \frac{1}{|I_\theta|} \sum_{k \in I_\theta} \xi_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} \xi_k \quad \text{where} \quad b_k = \frac{\mathbb{1}\{k \in I_\theta\}}{|I_\theta|} - \frac{\mathbb{1}\{k \in I_\theta^c\}}{|I_\theta^c|}.
$$

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Note that if $b_k = \dfrac{\mathbb{1}\{k \in I_\theta\}}{|I_\theta|} - \dfrac{\mathbb{1}\{k \in I_\theta^c\}}{|I_\theta^c|}$, then

$$\|b\|_1 = \sum_{k \in \mathbb{Z}^d} |b_k| = \sum_{k \in \mathbb{Z}^d} \left| \frac{\mathbb{1}\{k \in I_\theta\}}{|I_\theta|} - \frac{\mathbb{1}\{k \in I_\theta^c\}}{|I_\theta^c|} \right|$$

$$= \sum_{k \in I_\theta} \frac{\mathbb{1}\{k \in I_\theta\}}{|I_\theta|} + \sum_{k \in I_\theta^c} \frac{\mathbb{1}\{k \in I_\theta^c\}}{|I_\theta^c|} = 2$$

and

$$\|b\|_2^2 = \sum_{k \in \mathbb{Z}^d} |b_k|^2 = \sum_{k \in \mathbb{Z}^d} \left| \frac{\mathbb{1}\{k \in I_\theta\}}{|I_\theta|} - \frac{\mathbb{1}\{k \in I_\theta^c\}}{|I_\theta^c|} \right|^2$$

$$= \frac{1}{|I_\theta|} + \frac{1}{|I_\theta^c|} = \frac{|W|}{|I_\theta||I_\theta^c|}.$$

The main aim is to find a proper threshold y_α such that the probability of type I error of this statistic is less than α, which can be 5% or even 1%. Therefore, we need to find an upper bound P for the above probability and then set it smaller than 5%, which means,

$$\mathbb{P}\left(\max_{\theta \in \Theta_0} \left| \frac{1}{|I_\theta|} \sum_{k \in I_\theta} \xi_k - \frac{1}{|I_\theta^c|} \sum_{k \in I_\theta^c} \xi_k \right| \geq y_\alpha \right) \leq P = \alpha.$$

3. Main result

In this section, we find an upper bound of the above tail probability. Let $\left\{ \xi_k = (\xi_k^{(1)}, \xi_k^{(2)}, \ldots, \xi_k^{(n)}) \in \mathbb{R}^n, k \in \mathbb{Z}^d \right\}$ be a random field. Then, for any $W \in \mathbb{Z}^d, |W| < \infty$, one has

For $1 \leq p < \infty$,

$$\mathbb{P}\left\{ \left\| \sum_{k \in W} b_k \xi_k \right\|_p \geq y \right\} = \mathbb{P}\left\{ \sum_{i=1}^n \left| \sum_{k \in W} b_k \xi_k^{(i)} \right|^p \geq y^p \right\} \leq \sum_{i=1}^n \mathbb{P}\left\{ \left| \sum_{k \in W} b_k \xi_k^{(i)} \right| \geq \frac{y}{n^{1/p}} \right\}.$$

For $p = \infty$,

$$\mathbb{P}\left\{ \left\| \sum_{k \in W} b_k \xi_k \right\|_\infty \geq y \right\} = \mathbb{P}\left\{ \max_i \left| \sum_{k \in W} b_k \xi_k^{(i)} \right| \geq y \right\} \leq \sum_{i=1}^n \mathbb{P}\left\{ \left| \sum_{k \in W} b_k \xi_k^{(i)} \right| \geq y \right\}.$$

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Therefore, the upper bound for the tail inequality $\mathbb{P}\left\{\left\|\sum_{k\in W} b_k \xi_k\right\|_\infty \geq y\right\}$ can be found by bounding from above the quantity $\mathbb{P}\left\{\left|\sum_{k\in W} b_k \xi_k^{(i)}\right| \geq y\right\}$. Nevertheless, since

$$\mathbb{P}\left\{\left|\sum_{k\in W} b_k \xi_k^{(i)}\right| \geq y\right\} \leq \mathbb{P}\left\{\left|\sum_{k\in W} b_k \xi_k^{(i)}\right| \geq \frac{y}{n^{1/p}}\right\}$$

hence it is reasonable to bound from above the tail inequality for the case $p = \infty$. This can be established using ideas from [7] and [6], we obtain a bound of multivariate m-dependent random field as follows:

Theorem 1. Let $\left\{\xi_k = (\xi_k^{(1)}, \xi_k^{(2)}, \ldots, \xi_k^{(n)}) \in \mathbb{R}^n, k \in \mathbb{Z}^d\right\}$ be a mutivariate random field where $\{\xi_k^{(i)} \in \mathbb{R}, k \in W \subset \mathbb{N}^d\}$ is a stationary, m-dependent, real-valued random field. Assume there exists $H, \sigma > 0$ such that

$$\mathbb{E}[\xi_k^{(i)}]^\ell \leq \frac{p!}{2} H^{p-2}\sigma, \forall k \in W, i = 1, 2, \ldots n.$$

then

$$\mathbb{P}\left\{\left\|\sum_{k\in W} b_k \xi_k\right\|_\infty \geq y\right\} \leq 2n \exp\left\{\frac{-y^2}{4m^d\sigma^2} \frac{|I_\theta||I_\theta^c|}{|W|}\right\} \mathbb{1}\left\{|I_\theta^c| \leq \frac{\sigma^2|W|}{yH}\right\}$$

$$+ 2n \exp\left\{\frac{-y|I_\theta|}{2Hm^d} + \frac{\sigma^2|W||I_\theta|}{4H^2m^d|I_\theta^c|}\right\} \mathbb{1}\left\{|I_\theta^c| > \frac{\sigma^2|W|}{yH}\right\}.$$

Remark 1. In the case of univariate random field, i.e., $n = 1$, the upper bound coincides with the result from [6].

Theorem 2. Let $\left\{\xi_k = (\xi_k^{(1)}, \xi_k^{(2)}, \ldots, \xi_k^{(n)}) \in \mathbb{R}^n, k \in \mathbb{Z}^d\right\}$ be a mutivariate random field where $\{\xi_k^{(i)} \in \mathbb{R}, k \in W \subset \mathbb{N}^d\}$ is a stationary, m-dependent, real-valued random field. Assume there exists $H, \sigma > 0$ such that the inequality (6) holds, then

$$\mathbb{P}_{H_0}\left(\max_{\theta \in \Theta_0} |L(\theta)| \geq y_\alpha\right) \leq 2n \sum_{\theta \in \Theta_0, |I_\theta| \leq \frac{\sigma^2|W|}{yH}} \exp\left\{\frac{-y^2}{4m^d\sigma^2} \frac{|I_\theta||I_\theta^c|}{|W|}\right\}$$

$$+ 2n \sum_{\theta \in \Theta_0, |I_\theta| > \frac{\sigma^2|W|}{yH}} \exp\left\{\frac{-y|I_\theta|}{2Hm^d} + \frac{\sigma^2|W||I_\theta|}{4H^2m^d|I_\theta^c|}\right\}. \quad (3)$$

From (3), the null hypothesis will be rejected with type-I error controlled at level α if the critical value y is obtained by solving the following equation:

$$
2n \sum_{\theta \in \Theta_0, |I_\theta| \le \frac{\sigma^2 |W|}{yH}} \exp\left\{\frac{-y^2}{4m^d \sigma^2} \frac{|I_\theta||I_\theta^c|}{|W|}\right\}
$$

$$
+ 2n \sum_{\theta \in \Theta_0, |I_\theta| > \frac{\sigma^2 |W|}{yH}} \exp\left\{\frac{-y|I_\theta|}{2Hm^d} + \frac{\sigma^2 |W||I_\theta|}{4H^2 m^d |I_\theta^c|}\right\} = \alpha. \qquad (4)
$$

Remark 2. *If $\left\{\xi_k = (\xi_k^{(1)}, \xi_k^{(2)}, \dots, \xi_k^{(n)}) \in \mathbb{R}^n, k \in \mathbb{Z}^d\right\}$ is a multivariate Gaussian random field where $\mathbb{E}[\xi_0^{(i)}]^p \le \sigma^2, i = 1, \dots, n$, the choice $H = \sigma$ is suitable due to the fact that $\mathbb{E}|\xi_0^{(i)}|^p \le \sigma^{p-2}\sigma^2 \mathbb{E}|Z|^p, p \ge 2$ where $Z \sim \mathcal{N}(0,1)$.*

Remark 3. *By comparing the maximum of all statistics with a certain threshold, one can test the global hypothesis H_0, i.e., there exist anomalies within the whole random field. Furthermore, if for any $\theta \in \Theta$, we want to test the null hypothesis "$H_0(\theta)$: I_θ is homogeneous" versus "$H_1(\theta)$: I_θ contains anomalies", the upper bound serves as a useful tool to control the Family-wise error (FWER), where*

$$
FWER = \mathbb{P}(reject\ falsely\ at\ least\ one\ true\ hypothesis) = \mathbb{P}_{H_0}\left(\max_{\theta \in \Theta_0} |L(\theta)| \ge y_\alpha\right).
$$

4. Numerical results

In this section, we study the empirical distribution of the statistic $T_W(S)$ given in (2) and the behavior of the tail probability provided in (3) under different settings of σ and m.

Let $W = [1, P] \times [1, Q] \times [1, R] \bigcap \mathbb{N}^3$. The parametrized collection of scanning windows is defined as follows:

$$
\Theta_0 := \{\theta = (a_1, a_2, a_3, s_1, s_2, s_3) \in \mathbb{N}^6;
$$

$$
1 + a_1 + s_1 \le P, 1 + a_2 + s_2 \le Q, 1 + a_3 + s_3 \le R; \gamma_0 \le \frac{s_1 s_2 s_3}{W} \le \gamma_1\}.
$$

For each $\theta \in \Theta$, the scanning window $I_\theta = [1 + a_1, 1 + a_1 + s_1] \times [1 + a_2, 1 + a_2 + s_2] \times [1 + a_3, 1 + a_3 + s_3]$ is set within W. We choose $\gamma_0 = 0.05, \gamma_1 = 0.5$ to avoid the case that the scanning windows are either excessively small or excessively large relative to anomalies, where the scanning windows flatten out the statistical significance.

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We generate 500 realizations of a centered, m-dependent, multivariate Gaussian random field $\{\xi_k = (\xi_k^{(1)}, \xi_k^{(2)}, \xi_k^{(3)}) \in \mathbb{R}^3, k \in W\}$, where $W = [1,50] \times [1,50] \times [1,50], \xi_k^{(i)} \sim \mathcal{N}(0,1), k \in W, i = 1,2,3$. With $m = 5,7$, the m-dependence assumption can be achieved by setting $Y_{1+m\ell}, \ell \in W$ are independent and $Y_{1+m\ell} = Y_{r+m\ell}, r \in \{1, \ldots, m\}^3$. The set of scanning windows $\{I_\theta, \theta \in \Theta_0\}$ is defined where all I_θ are equal cubic of the size 30×30, satisfying the requirements of the parametric setting. In this case, $|\Theta_0| = 9261$.

	$\sigma^2 = 0.5$	$\sigma^2 = 0.6$	$\sigma^2 = 0.7$	$\sigma^2 = 0.8$	$\sigma^2 = 0.9$	$\sigma^2 = 1.0$	$\sigma^2 = 1.1$
$m = 3$	0.6009	0.6424	0.6814	0.7183	0.7533	0.7868	0.8191
$m = 4$	0.8398	0.8978	0.9522	1.0038	1.0528	1.1001	1.1459
$m = 5$	1.1861	1.2675	1.3490	1.4305	1.5120	1.5935	1.6750
$m = 6$	1.5693	1.6987	1.8281	1.9575	2.0869	2.2163	2.3457
$m = 7$	2.0794	2.2725	2.4656	2.6588	2.8519	3.0450	3.2381
$m = 8$	2.7342	3.0092	3.2842	3.5593	3.8343	4.1093	4.3843
$m = 9$	3.5521	3.9293	4.3066	4.6838	5.0610	5.4382	5.8155
$m = 10$	4.5230	5.0217	5.5205	6.0193	6.5180	7.0168	7.5156

Table 1: Critical value y from 4 with $\alpha = 0.05$

Let $\alpha = 0.05$, we compare the empirical critical value $\hat{y}_{0.05}$ with $y_{0.05}$ computed from Equation (3). For $m = 5$ and $m = 7$, one has $\hat{y}_{0.05} = 0.5369$ and $\hat{y}_{0.05} = 0.7259$, respectively. The critical values $y_{0.05}$ with different values of m and σ are shown in Table 1. For instance, since the exact values of m are known, one has the theoretical critical values $y_{0.05} = 1.5935$ and $y_{0.05} = 2.8519$, leading the test to be conservative. However, from Table 1, the theoretical critical value is quite sensitive to any changes of m, suggesting to choose a smaller m in the computation of $y_{0.05}$, such as $m = 3$, resulting in $y_{0.05} = 0.7868$, therefore controlling the type-I error at the desired level $\alpha = 0.05$.

When m is unknown, it can be estimated by determining the covariance function of the random field. The value of m should be selected such that the empirical covariance function is sufficiently close to zero.

5. Conclusion

In this paper, we generalized the results given in [6] for the multi-dimensional case of a random field. The results show that by properly choosing the value of m, one is able to test the global hypothesis H_0, i.e., if the observed random field contains anomalies, at the pre-determined significant level α. The main challenge

of this approach is that one has to employ the Bonferroni correction [3] due to the dependence between observations from different scanning windows, which usually makes the test extremely conservative, especially when $|\Theta_0|$ is large.

Nevertheless, in the context of multiple hypotheses testing, the obtained upper bound for the tail probability provides a global threshold for each statistic, controlling the Family-Wise error rate at the significance level α, allowing one to localize anomalies within a random field.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] P. Alonso Ruiz, E. Spodarev, Entropy-based Inhomogeneity Detection in Fiber Materials, Methodology and Computing in Applied Probability, 20 (2018) 1223–1239, https://doi.org/10.1007/s11009-017-9603-2.

[2] P. Alonso-Ruiz, E. Spodarev, Estimation of entropy for Poisson marked point processes, Advances in Applied Probability, 49 (2017) 258–278, http://www.jstor.org/stable/44985422.

[3] C. E. Bonferroni, Teoria Statistica delle classi e calcolo delle probabilita, Pubblicazioni Del R Istituto Superiore Di Scienze Economiche E Commericiali Di Firenze, 1936.

[4] B. E. Brodsky, B. S. Darkhovsky, Nonparametric Methods in Change Point Problems, Springer Dordrecht, 1993.

[5] B. Brodsky, B. Darkhovsky, Non-parametric statistical diagnosis: problems and methods, Vol. 509, Springer Science & Business Media, 2013.

[6] D. Dresvyanskiy, T. Karaseva, V. Makogin, S. Mitrofanov, C. Redenbach, E. Spodarev, Detecting anomalies in fibre systems using 3-dimensional image data, Statistical Computing, 30 (2020) 817–837, https://doi.org/10.1007/s11222-020-09921-1.

[7] L. Heinrich, Some Bounds of Cumulants of m-Dependent Random Fields, Mathematische Nachrichten, 149 (1) (1990) 303–317, https://doi.org/10.1002/mana.19901490123.

[8] R. H. Jones, D. H. Crowell, L. E. Kapuniai, Change Detection Model for Serially Correlated Multivariate Data, Biometrics, 26 (2) (1970) 269–280, https://doi.org/10.2307/2529074.

[9] E. Page, Continuous Inspection Schemes, Biometrika, 41 (1/2) (1954) 100–115, https://doi.org/10.2307/2333009.

[10] N. Peach, M. Basseville, and I. Nikiforov, Detection of Abrupt Changes: Theory and Applications, Journal of The Royal Statistical Society Series A-statistics in Society, 158 (1) (1995) 185–186, https://doi.org/10.2307/2983416.

[11] D. Siegmund, Sequential Analysis: Tests and Confidence Intervals, 1st ed., Springer Series in Statistics, Springer Science+Business Media New York 1985, XI, 274 pp., https://doi.org/10.1007/978-1-4757-1862-1.

[12] A. Tartakovsky, Rapid Detection of Attacks in Computer Networks by Quickest Changepoint Detection Methods, in: Data Analysis for Network Cyber-Security, Chapter 2, pp. 33-70, https://doi.org/10.1142/9781783263752_0002.

[13] A. Tartakovsky, I. Nikiforov, M. Basseville, Sequential Analysis: Hypothesis Testing and Changepoint Detection, CRC Press, 2014.

[14] R. Zhang, Statistical analysis of the non-stationary binomial AR(1) model with change point, Applied Mathematical Modelling, 118 (2023) 152–165, https://doi.org/10.1016/j.apm.2023.01.032.