1. a) $\displaystyle\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{4}\int (1-x^4)'\,(1-x^4)^{-\frac{1}{2}} = -\frac{1}{2}\sqrt{1-x^4}$

b) $\displaystyle\int \frac{3x^2+1}{x(x-1)(x+1)}\,dx = \int -\frac{1}{x} + \frac{2}{x-1} + \frac{2}{x+1}\,dx$

$= 2\ln(x-1) + 2\ln(x+1) - \ln x$

c) $\displaystyle\int x \arctan x\,dx = \frac{1}{2}\int (x^2+1)' \arctan x\,dx$

$\displaystyle = -\frac{1}{2}\int \frac{(x^2+1)}{(x^2+1)}\,dx + \frac{1}{2}(x^2+1)\arctan x = \frac{1}{2}\left((x^2+1)\arctan x - x\right)$

d) $\displaystyle (x^4+3x^3+2x-4):(x^2-9) = (x^2+3x+9) + \frac{29x+77}{(x-3)(x+3)}$

$-(x^4-9x^2)$

$3x^3+9x^2+2x-4$

$-(3x^3-27x)$

$9x^2+29x-4$

$-(9x^2-81)$

$29x+77$

$\displaystyle a_1 = \frac{29\cdot 3+77}{6} = \frac{164}{6} = 27\tfrac{1}{3}$

$\displaystyle a_2 = \frac{29\cdot(-3)+77}{-6} = \frac{10}{6} = \frac{5}{3}$

$\displaystyle\int \frac{x^4+3x^3+2x-4}{x^2-9}\,dx = \int x^2+3x+9\,dx + \int \frac{29x+77}{(x-3)(x+3)}\,dx$

$\displaystyle = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 9x + \int \frac{a_1}{(x-3)} + \frac{a_2}{x+3}\,dx$

$\displaystyle = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 9x + 27\tfrac{1}{3}\ln(x-3) + \frac{5}{3}\ln(x+3)$

2. $x^2+y^2=16$, $x^2+(y-5)^2=9$

$x^2=16-y^2$, $x^2=9-y^2+10y-25=-y^2+10y-16$

Schnittpunkte: $16-\cancel{y^2}=-\cancel{y^2}+10y-16 \Rightarrow y=\frac{32}{10}=\frac{16}{5}$

$\rightarrow x^2=16-\frac{256}{25}=\frac{144}{25} \rightarrow x_{1/2}=\pm\frac{12}{5}$

$\Rightarrow$ Fläche $=2\int_0^{12/5}\left(\sqrt{16-x^2}-\left(5-\sqrt{9-x^2}\right)\right)dx$

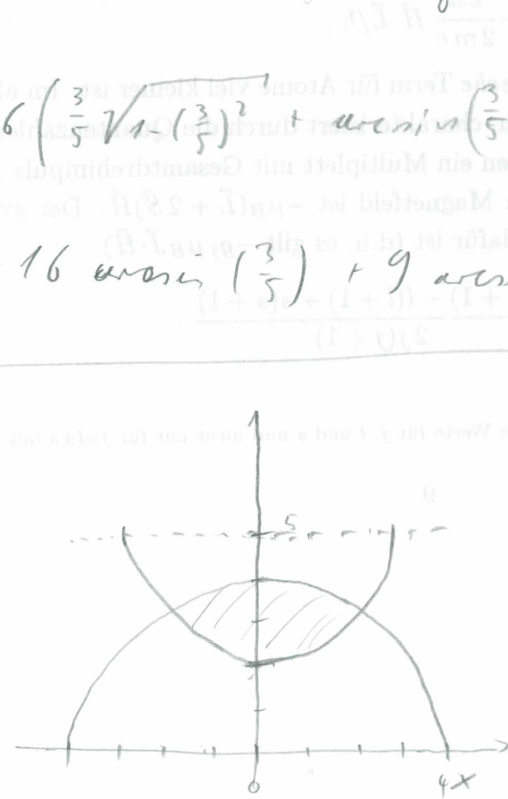
$=2\left(\int_0^{12/5}-5\,dx+4\int_0^{12/5}\sqrt{1-\left(\frac{x}{4}\right)^2}\,dx+3\int_0^{12/5}\sqrt{1-\left(\frac{x}{3}\right)^2}\,dx\right)$

$=2\left(-12+16\int_0^{3/5}\sqrt{1-y^2}\,dy+9\int_0^{4/5}\sqrt{1-y^2}\,dy\right)$ ($\frac{x}{4}=y$, $\frac{x}{3}=y$)

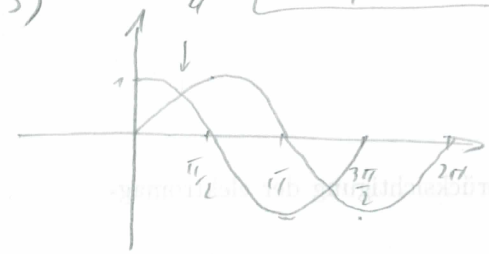
$=2\left(-12+\frac{16}{2}\left(y\sqrt{1-y^2}+\arcsin y\right)\Big|_0^{3/5}+\frac{9}{2}\left(y\sqrt{1-y^2}+\arcsin y\right)\Big|_0^{4/5}\right)$

$=-24+16\left(\frac{3}{5}\sqrt{1-\left(\frac{3}{5}\right)^2}+\arcsin\left(\frac{3}{5}\right)\right)+9\left(\frac{4}{5}\sqrt{1-\left(\frac{4}{5}\right)^2}+\arcsin\left(\frac{4}{5}\right)\right)$

$=-12+16\arcsin\left(\frac{3}{5}\right)+9\arcsin\left(\frac{4}{5}\right)\approx 6{,}64162$



3) $\boxed{\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}}$



Fläche $= \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$

$$= -\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4} + \cos\frac{\pi}{4} + \sin\frac{\pi}{4}$$

$$= \frac{4}{\sqrt{2}} = \sqrt{8}$$

4) $$\mathrm{Vol} = \pi\int_0^{\pi/2} \left(e^{-x}\sqrt{\sin x}\right)^2 dx = \pi\int_0^{\pi/2} e^{-2x}\sin x\,dx$$

[NR: $\int e^{-2x}\sin x\,dx = -2\int e^{-2x}\cos x - e^{-2x}\cos x$

$= 4\int e^{-2x}\sin x - 2e^{-2x}\sin x - e^{-2x}\cos x$

$\Rightarrow \int e^{-2x}\sin x = -\frac{1}{5}e^{-2x}(2\sin x + \cos x)$]

$$= \frac{\pi}{5}\left(-2e^{-\pi} + 1\right) \approx 0.5740$$