

$$1. a) \int \frac{x^3}{\sqrt{1-x^4}} dx = -\frac{1}{4} \int (1-x^4)^{-\frac{1}{2}} (1-x^4)' dx = -\frac{1}{2} \sqrt{1-x^4}$$

1. Mittelwertsatz für stetige Funktionen aus Intervall

$$b) \int \frac{3x^2+1}{x(x-1)(x+1)} dx = \int \frac{1}{x} + \frac{2}{x-1} - \frac{2}{x+1} dx$$

$$= 2 \ln|x-1| + 2 \ln|x+1| - \ln|x|$$

Basierend auf der Sichtweise des Potenzintegrals der Potenzintegrale ist folgendes zu zeigen:

$$c) \int x \arctan x dx = \frac{1}{2} \int (x^2+1) \arctan x dx$$

$$= -\frac{1}{2} \int \frac{(x^2+1)}{(x^2+1)} dx + \frac{1}{2} (x^2+1) \arctan x = \frac{1}{2} ((x^2+1) \arctan x - x)$$

$$d) \frac{(x^4+3x^3+2x-4)}{(x^2-9)} = (x^2-9) \cdot (x^2+3x+9) + \frac{29x+77}{(x-3)(x+3)}$$

$$-\frac{(x^4+9x^2)}{3x^3+9x^2+2x-4}$$

$$-(3x^3-27x)$$

$$\frac{9x^2+29x-4}{29x+77}$$

$$-\frac{(9x^2-81)}{29x+77}$$

$$u_1 = \frac{29 \cdot 3 + 77}{6} = \frac{144}{6} = 24 \frac{1}{3}$$

$$u_2 = \frac{29 \cdot (-3) + 77}{-6} = \frac{16}{-6} = -\frac{8}{3}$$

$$\int \frac{x^4+3x^3+2x-4}{x^2-9} dx = \int x^2+3x+9 dx + \int \frac{29x+77}{(x-3)(x+3)} dx$$

$$= \frac{1}{3}x^3 + \frac{3}{2}x^2 + 9x + \int \frac{u_1}{(x-3)} + \frac{u_2}{x+3} dx -$$

$$= \frac{1}{3}x^3 + \frac{3}{2}x^2 + 9x + 24 \frac{1}{3} \ln|x-3| + \frac{5}{3} \ln|x+3|$$

(1)

$$2. \quad x^2 + y^2 = 16 \quad , \quad x^2 + (y-5)^2 = 9$$

$$\therefore x^2 = 16 - y^2 \quad | \quad x^2 = 9 - y^2 + 10y - 25 = -y^2 + 10y - 16$$

Die Gleichung ist ein Kreis um  $(0, 5)$  mit dem Radius  $4$ . Die Gleichung ist ein Kreis um  $(0, 5)$  mit dem Radius  $3$ .

$$\text{Schnittpunkte: } 16 - y^2 = -y^2 + 10y - 16 \Rightarrow 10y = \frac{32}{10} = \frac{16}{5}$$

$$(0) \quad [x \times 5] \cdot 5 - 5 \cdot 5 + 5 \cdot y = 5 \cdot x + 5 \cdot y = (5 \cdot 5) \underline{(5 \cdot 5)}$$

$$\rightarrow x^2 = 16 - \frac{256}{25} = \frac{144}{25} \quad | \quad x_{1/2} = \pm \frac{12}{5}$$

$$\Rightarrow \text{Fläche} = 2 \int_0^{12/5} \left( \sqrt{16-x^2} - (5 - \sqrt{9-x^2}) \right) dx$$

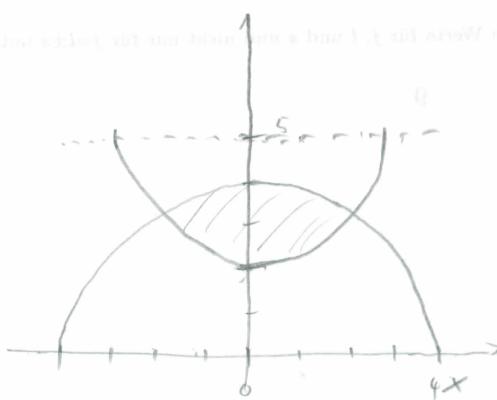
$$(0) \quad \int_0^{12/5} \left( -5 \sqrt{x} + 4 \int_0^{12/5} \sqrt{1-(\frac{x}{4})^2} \sqrt{x} + 3 \int_0^{12/5} \sqrt{1-(\frac{x}{3})^2} \right) dx$$

$$= 2 \left( -12 + 16 \int_0^{3/5} \sqrt{1-y^2} dy + 9 \int_0^{4/5} \sqrt{1-y^2} dy \right)$$

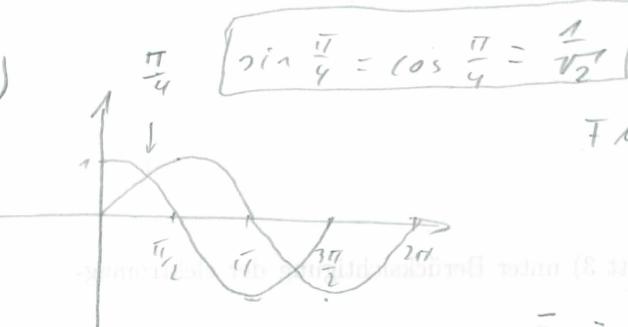
$$= 2 \left( -12 + 16 \left[ \frac{3}{5} \sqrt{1-y^2} + \arcsin(y) \right] \Big|_0^{3/5} + 9 \left[ \frac{4}{5} \sqrt{1-y^2} + \arcsin(y) \right] \Big|_0^{4/5} \right)$$

$$= -24 + 16 \left( \frac{3}{5} \sqrt{1-(\frac{3}{5})^2} + \arcsin(\frac{3}{5}) \right) + 9 \left( \frac{4}{5} \sqrt{1-(\frac{4}{5})^2} + \arcsin(\frac{4}{5}) \right)$$

$$= -12 + 16 \arcsin(\frac{3}{5}) + 9 \arcsin(\frac{4}{5}) \approx 6,64162$$



3)



$$\text{Fläche} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx$$

 $\frac{\pi}{4}$  $\frac{\pi}{2}$  $\frac{3\pi}{4}$  $\pi$ 

zu Diagonalenpunkt

Wegen der linearen Diagonalen (siehe 3. Aufgabe) mit folgenden Werten

des ersten Potenzialen V und folgende Gleichung

$$= -\cos \frac{5\pi}{4} - \sin \frac{5\pi}{4} + \omega \cdot \frac{\pi}{4} + \theta \sin \frac{\pi}{4}$$

(1)

$$V_0 + \frac{1}{2} m \omega^2 + (\sqrt{h} s - \theta) \cdot \theta \omega = \frac{1}{4} \pi$$

$$=\frac{4}{\sqrt{2}} = \sqrt{8}$$

4)

$$\left( \begin{array}{cc} 0 & \sqrt{3}s - \theta \omega \\ \sqrt{3}s - \theta \omega & 0 \end{array} \right) + \left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \Delta H$$

$$V_0 l = \pi \int_0^{\frac{\pi}{2}} (e^{-x} \sqrt{\sin x})^2 dx = \pi \int_0^{\frac{\pi}{2}} e^{-2x} \sin x dx$$

$$\left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) = I, \quad \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right) = \theta I$$

$$\left[ NR : \int e^{-2x} \sin x dx = -2 \int e^{-2x} \cos x = e^{-2x} \cos x \right]$$

$$\Rightarrow \int e^{-2x} \sin x dx = -\frac{1}{2} e^{-2x} (2 \sin x + \cos x)$$

$$\left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \theta I (N \omega - \theta \omega - 3) \quad \left( \begin{array}{cc} h \omega & 0 \\ 0 & -\theta \end{array} \right) = \left( \begin{array}{cc} +\theta & 0 \\ 0 & -\theta \end{array} \right) \theta I = \left( \begin{array}{cc} +\theta & 0 \\ 0 & -\theta \end{array} \right) \theta$$

$$= \frac{\pi}{5} \left( -2 e^{-\pi} + 1 \right) \approx 0,57490$$

$$\left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \frac{1}{\theta} \left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & 1 \end{array} \right) = -I$$

so dass  $x = 0$  der eigene Eigenwert von (3) ist, also nur eine

$$\left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \theta I (N \omega - \theta \omega - 3)$$

$$\left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \theta I (N \omega - \theta \omega - 3) = \theta I (N \omega - \theta \omega - 3)$$

$$\left( \begin{array}{cc} h \omega - \theta \omega & 0 \\ 0 & (h \omega - \theta \omega) \cdot \theta \end{array} \right) = \theta I (N \omega - \theta \omega - 3)$$