Capital Requirement for German Unit-Linked Insurance Products

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Abstract

Innovative life insurance products such as unit-linked life insurance, hybrid life insurance, and variable annuities are rapidly gaining popularity and becoming a major part of new business in Germany. However, since traditional life insurance products still dominate the portfolios of life insurance companies, discussions about the standard formula for determining the solvency capital requirement have focused on this type of business. Any detailed discussion on how to calculate the solvency capital requirement for innovative life insurance products within the standard formula has yet to occur.

This paper brings to light some interesting facts about unit-linked business and Solvency II. In particular, it analyzes the impact of the transition from Solvency I to Solvency II on the solvency capital requirement of a German unit-linked insurance product with guaranteed death benefits. The modeling of lapses is another focus of research, reflecting the increased importance of lapse risks for innovative life insurance products. Since there are strong concerns about nonlinearities between the various risks, especially between market risk and lapse risk, the paper examines this problem as well. Finally, an alternative method for calculating the net solvency capital requirement, the so-called single equivalent scenario (also referred to as the killer-scenario), is presented.

Keywords: Unit-linked insurance, Solvency II, standard formula, single equivalent scenario, dynamic policyholder behavior

1 Introduction

Innovative life insurance products have been gaining in popularity during the last decade and now represent nearly a third of new business in Germany (see Helfenstein & Barnshaw (2003), Enz (2006) and Märten & Daalmann (2009)). However, and despite the importance of these products to the future of the insurance industry, most discussions about the Solvency II framework focus on traditional insurance products. The results of the last quantitative impact study, QIS4, indicate that most insurance companies do not calculate the solvency capital requirement for innovative life insurance products as systematically as they do for traditional products (see CEIOPS (2008a)).

In academic literature of the last decade, fair valuation of life insurance products has been an emerging field. Especially traditional life insurance contracts with interest rate guarantees have been analyzed in particular (see Bauer *et al.* (2006), Bacinello (2001) and Steffensen (2002)). Common options of traditional policies such as the option to surrender also draw growing attention (see Grosen & Jorgensen (2000) and Steffensen (2002)). Furthermore, the recent low interest period, changing customer need and tax law led to increased new business of unit-linked life insurance, hybrid life insurance and variable annuities and therefore a development of pricing techniques (see Bauer *et al.* (2008)). However, innovative life insurance products still need to be examined in conjunction with recent regulatory changes in Europe (e.g. Solvency II).

The aim of this paper is to provide the first contribution to a discussion about the solvency capital requirement for innovative life insurance products. The paper brings together fair valuation, risk analysis and a detailed product design and should be of interest to academics as well as to practitioners.

Innovative life insurance products differ from traditional life insurance products in some fundamental aspects and therefore require an in-depth risk examination. For both insurers and policyholders, the value of an innovative life insurance product is expected to be somewhat volatile, since the capital is mostly invested in risky assets, compared to the fixed-income-oriented investment strategies of traditional life insurance products. Furthermore, innovative life insurance products are usually complex in their structure and contain a broad range of options and guarantees (see Gatzert (2009)). These insurance products also induce dynamic policyholder behavior. Their volatile value, complexity, dynamic policyholder behavior, options, and guarantees can all have an unexpected influence on the solvency capital requirement.

The contribution of this paper is to identify the main risks of a unit-linked life insurance product and to discuss two methods for calculating the solvency capital requirement, the standard formula and the single equivalent scenario. Furthermore, a way to implement dynamic policyholder behavior in the standard formula is presented. This paper also provides a methodology for calculating solvency capital requirement for other innovative life insurance products.

The paper is organized as follows: To establish a methodology, a German unit-linked insurance with guaranteed death benefits is examined based on the standard formula method (see CEIOPS (2008c)). A simplified version of the standard formula and information about the calculation of the solvency capital requirement is presented in chapter 2, the stress scenarios can be found in capter 5. The product design is illustrated in chapter 3 and includes fixed and variable costs, mortality, deterministic and dynamic lapses, kickbacks, a bonus system in accordance with German law, and a realistic set of parameters. The analysis is performed on products with single premiums as well as on products with regular premiums. Furthermore, the paper analyzes the impact of the transition from Solvency I (see Müller (1997) and Bundesministerium der Justiz (2009)) to Solvency II on the solvency capital requirement for the unit-linked insurance product with different death benefits and different sets of parameters. Chapter 4 provides a detailed description of the asset and the liability models and their implementation. A method for modeling dynamic lapses is discussed in chapter 7. The single equivalent scenario is presented in chapter 8. The paper examines the linearity assumption by means of a sensitivity analysis in chapter 6.

2 Solvency capital requirement

The solvency capital of an insurance company guarantees the solvability of the latter during a financial distress. Regarding the importance of insurance to the society, economy and public welfare, the insurance company should have enough capital to overcome almost every crisis. In Solvency I the solvency capital requirement is calculated via a factor-based framework. This framework is easy to understand and easy to use, and it requires only some balance sheet values and the corresponding risk factors. The Solvency I capital requirement for German unit-linked products, where the policyholder bears the investment risk, is defined as 1% of the investment fund value plus 0.3% of the risk premium[1]. The downside of a factor-based framework is that it does not reflect the actual risks. Solvency II, as a risk based framework, will provide a more sophisticated view on the risk taking of an insurance company[2].

In the Solvency II framework, the amount of solvency capital an insurance company has to hold is in the broader sense defined as the amount of capital needed to survive a "one in two hundred years crisis".

2.1 SCR - the mathematical approach

Let X denote a risk, the solvency capital requirement of $SCR_\alpha(X)$ is defined as

$$SCR_\alpha(X) = VaR_\alpha(X) - \mathbb{E}[X] \tag{1}$$

The following transformations lead to a mathematical definition of the SCR[3]:

$$\begin{aligned} SCR_\alpha(X) &= VaR_\alpha(X) - \mathbb{E}[X] && (2)\\ &= VaR_\alpha(X - \mathbb{E}[X]) && (3)\\ &= \underset{x}{\operatorname{argmin}} \ (P\left[X - \mathbb{E}[X] \le x\right] \ge \alpha) && (4)\\ &= \underset{x}{\operatorname{argmin}} \ (1 - P\left[X - \mathbb{E}[X] > x\right] \ge \alpha) && (5)\\ &= \underset{x}{\operatorname{argmin}} \ (P\left[X - \mathbb{E}[X] > x\right] \le 1 - \alpha) && (6) \end{aligned}$$

[1]See Bundesministerium der Justiz (2009) and Müller (1997).

[2]See Doff (2008), Duverne & Le Douit (2009), Holzmüller (2009), Elderfield (2009) or Steffen (2008) for a comparison of different regulatory frameworks and general information about Solvency II.

[3]As introduced in Bauer *et al.* (2009). Bergmann's notion is used for practical applications. It is approximately equivalent to $P(AC_1 \geq 0 | AC_0 = x) \geq \alpha$, but avoids the implicit nature of the definition.

Now given a one in two hundred years crisis and a one year horizon and X set to $X = -\frac{AC_1}{(1+i)}$ with $\mathbb{E}[X] = -AC_0$, the solvency capital requirement SCR can be expressed as[4]

$$SCR_\alpha = \underset{x}{\operatorname{argmin}} \left(P\left[AC_0 - \frac{AC_1}{(1+i)} > x \right] \leq 1 - \alpha \right) \tag{7}$$

for an $\alpha = 0.995$, an interest rate i and the available capital AC_t in time $t = 0$ and $t = 1$[5].

2.2 SCR - the standard formula

Although the formula above perfectly defines the solvency capital requirement, it is not practical because of two reasons: firstly, it is very difficult to describe an insurance company as a whole with a stochastic model and secondly, nested simulations are needed. In order to provide a more simple approach, especially for small insurance companies that do not use an internal model, CEIOPS introduced the standard formula. The main simplification is the definition of deterministic stress scenarios that should represent the one in two hundred years crisis. In addition, risks are supposed to be multivariate normally distributed. Let $X = -\Pi$ denote a random loss variable or negative profits Π, then the SCR can be simplified to[6]

$$SCR = VaR(-\Pi) - \mathbb{E}[-\Pi] \tag{8}$$

$$SCR = (\mathit{Liabilities} - \mathit{Assets})\,|_{stress} - (\mathit{Liabilities} - \mathit{Assets}) \tag{9}$$

$$SCR = (\mathit{Assets} - \mathit{Liabilities}) - (\mathit{Assets}|_{stress} - \mathit{Liabilities}|_{stress}) \tag{10}$$

The stress scenarios are formulated for various risk modules (interest rates, equity, mortality, lapses and expenses) and are aggregated via a correlation matrix. Let X_i denote the loss variable exposed to a risk i defined in a risk module and $SCR\,(X_i)$ denote the solvency capital requirement calculated for the same risk module.

[4] Implicitly assuming that dividends have not been paid to shareholders yet at $t = 1$.

[5] The available capital can be expressed in terms of MCEV. See Bauer *et al.* (2009) for more information.

[6] *Assets* and *Liabilities* denote the expected present value of all Assets and Liabilities as defined in QIS4 (see below for more information).

Then the aggregated solvency capital requirement $SCR(X)$ for the aggregated loss variable[7] $X = \sum_i X_i$ is defined as[8]:

$$\begin{aligned} SCR_\alpha(X) &= VaR_\alpha(X) - \mathbb{E}[X] && (11)\\ &= VaR_\alpha\left(\sum_i X_i\right) - \mathbb{E}[X] && (12)\\ &= \sqrt{\sum_{i,j} \rho_{i,j} \left(VaR_\alpha(X_i) - \mathbb{E}[X_i]\right)\left(VaR_\alpha(X_j) - \mathbb{E}[X_j]\right)} && (13)\\ &\qquad + \mathbb{E}\left[\sum_i X_i\right] - \mathbb{E}[X] \\ &= \sqrt{\sum_{i,j} \rho_{i,j} SCR_\alpha(X_i) SCR_\alpha(X_j)} && (14) \end{aligned}$$

Figure 1 shows a simplified modular view on the standard formula. Only relevant risks for a German unit-linked insurance product are considered.

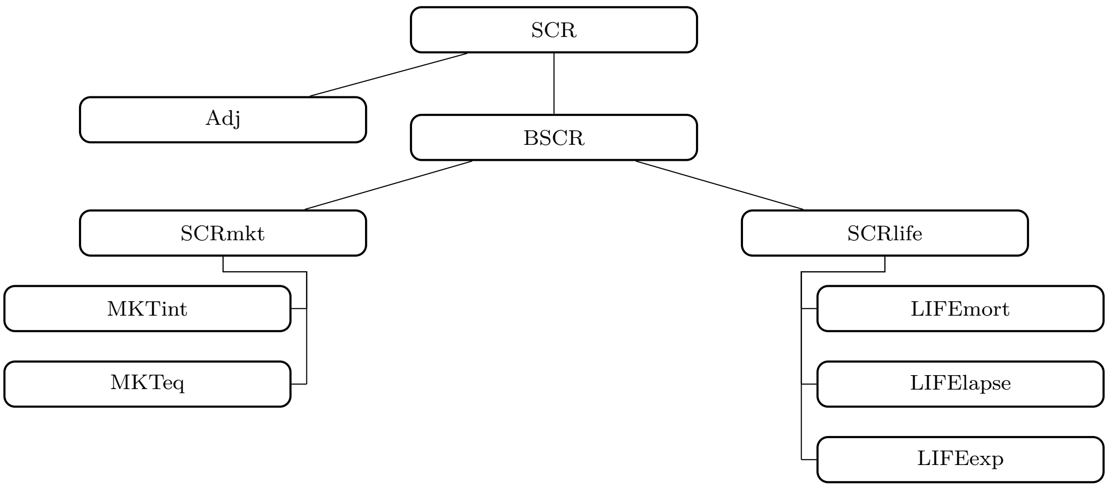


Figure 1: Modular structure of the SCR

[7]With $\mathbb{E}[X] = \mathbb{E}\left[\sum_i X_i\right]$.

[8]See GDV (2005, page 88-93).

The solvency capital requirement can be expressed with the following formulae[9]:

$$SCR = \sqrt{SCR_{mkt}^2 + 2 \cdot \rho_{mkt,life} \cdot SCR_{mkt} SCR_{life} + SCR_{life}^2} \tag{15}$$

$$SCR_{mkt} = \sqrt{SCR_{int}^2 + 2 \cdot \rho_{int,eq} \cdot SCR_{int} SCR_{eq} + SCR_{eq}^2} \tag{16}$$

$$SCR_{life} = \sqrt{SCR_{mort}^2 + SCR_{lapse}^2 + SCR_{exp}^2 + 2 \cdot \rho_{mort,lapse} \cdot SCR_{mort} SCR_{lapse} + 2 \cdot \rho_{mort,exp} \cdot SCR_{mort} SCR_{exp} + 2 \cdot \rho_{lapse,exp} \cdot SCR_{lapse} SCR_{exp}} \tag{17}$$

The corresponding correlation factors can be obtained from table 1.

CorrSCR=	SCRmkt	SCRlife
SCRmkt	1	0.25
SCRlife	0.25	1

CorrMkt=	SCRint	SCReq
SCRint	1	0
SCReq	0	1

CorrLife=	SCRmort	SCRlapse	SCRexp
SCRmort	1	0	0.25
SCRlapse	0	1	0.5
SCRexp	0.25	0.5	1

Table 1: Correlation matrices

[9]See chapter 5 for details on the relevant risk modules.

According to the principles of Solvency II, a "best estimate is equal to the probability-weighted average of future cash-flows, taking account of the time value of money, using the relevant risk-free interest rate term structure. The calculation of best estimate should be based upon current and credible information and realistic assumptions and be performed using adequate actuarial methods and statistical techniques."[10]. In this case, the best estimate of technical provisions equals the best estimate of liabilities. In order to simplify the task, a risk margin will not be calculated. The valuation of assets is performed with a mark to model procedure. This framework ensures a market-consistent valuation of all assets and liabilities. Let $\Pi = Assets - Liabilities$ denote the value of an insurance policy. Then, the solvency capital requirement for the particular risk modules is defined as

$$SCR_{int-up} = (\Pi) - (\Pi|_{up-shock}) \tag{18}$$

$$SCR_{int-down} = (\Pi) - (\Pi|_{down-shock}) \tag{19}$$

$$SCR_{int} = \max\left(SCR_{int-up}, SCR_{int-down}; 0\right) \tag{20}$$

$$SCR_{eq} = \max\left((\Pi) - (\Pi|_{eqshock}); 0\right) \tag{21}$$

$$SCR_{mort} = \max\left((\Pi) - (\Pi|_{mortshock}); 0\right) \tag{22}$$

$$SCR_{lapse-up} = (\Pi) - (\Pi|_{up-shock}) \tag{23}$$

$$SCR_{lapse-down} = (\Pi) - (\Pi|_{down-shock}) \tag{24}$$

$$SCR_{lapse-mass} = (\Pi) - (\Pi|_{mass-shock}) \tag{25}$$

$$SCR_{lapse} = \max\left(SCR_{lapse-up}; SCR_{lapse-down}; SCR_{lapse-mass}; 0\right) \tag{26}$$

$$SCR_{exp} = \max\left((\Pi) - (\Pi|_{expshock}); 0\right) \tag{27}$$

2.3 The risk absorbing effect of future profit sharing

Future bonuses paid out to the policyholders will change while calculating the profits under a stress scenario when stochastic profit sharing rules are used. The solvency capital calculated with adjusted bonuses is referred to as the net solvency capital requirement ($nSCR$). The solvency capital calculated with constant bonuses through a stress is referred to as the basic solvency

[10] See CEIOPS (2008c, page 13-14).

capital requirement ($BSCR$). The value of the future discretionary bonuses FDB can be defined as

$$FDB = \Pi|_{\text{no profit sharing}} - \Pi|_{\text{profit sharing}} \tag{28}$$

The adjustment for the risk absorbing effect of future profit sharing to the BSCR is then defined as

$$Adj_{FDB} = \min\left(BSCR - nSCR, FDB\right) \tag{29}$$

and the overall SCR or net basic solvency capital requirement $nBSCR$ is defined as

$$SCR = nBSCR = BSCR - Adj_{FDB} \tag{30}$$

The calculation of the $BSCR$ is performed with "constant" bonuses throughout all stress scenarios. There are several interpretations what "constant" bonuses are. One interpretation is that the $BSCR$ "should be calculated under the condition that the absolute amount of future discretionary benefits cash flows per policy and year remain unchanged before and after the shock being tested"[11]. This direct calculation of the $BSCR$ requires storage of bonuses for every simulation step and every simulation path. In order to avoid a huge computational capacity requirement and improve the practicability, the problem can be simplified using an alternative interpretation:
The calculation of the $BSCR$ is performed with a "constant value" of bonuses. Therefore, the $BSCR$ is "calculated under the condition that the value of future discretionary benefits remains unchanged before and after the shock being tested"[12]. Let $Liabilities = Bonuses + otherLiabilities$ be a decomposition of the liabilities, then Π and $BSRC$ can be defined as

$$\begin{align} \Pi &= Assets - Bonuses - otherLiabilities \tag{31}\\ BSCR &= (Assets - Bonuses - otherLiabilities) \tag{32}\\ &\quad - (Assets|_{stress} - Bonuses|_{stress} - otherLiabilities|_{stress}) \nonumber \end{align}$$

Since the bonuses should be constant in order to calculate the $BSCR$ and therefore $Bonuses = Bonuses|_{stress}$, the above equation can be simplified to

$$\begin{align} BSCR &= (Assets - otherLiabilities) \tag{33}\\ &\quad - (Assets|_{stress} - otherLiabilities|_{stress}) \nonumber \end{align}$$

[11]See CEIOPS (2009b).
[12]See CEIOPS (2009b).

The above $BSCR$ corresponds to the $nSCR$ calculated without any profit sharing. Therefore, in order to calculate the $BSCR$ the profit sharing parameters "risk profit participation rate" (rb^{rate}) and "expense profit participation rate" (cb^{rate}) are set to 0. In general, with participation rates other than zero, the $nSCR$ is defined as:

$$\begin{aligned} nSCR = {} & (Assets - Bonuses - otherLiabilities) \\ & - (Assets|_{stress} - Bonuses|_{stress} - otherLiabilities|_{stress}) \end{aligned} \tag{34}$$

with

$$Bonuses \neq Bonuses|_{stress} \tag{35}$$

Figures 2 and 3 show different solvency balance sheets for $BSCR$ and $nSCR$.

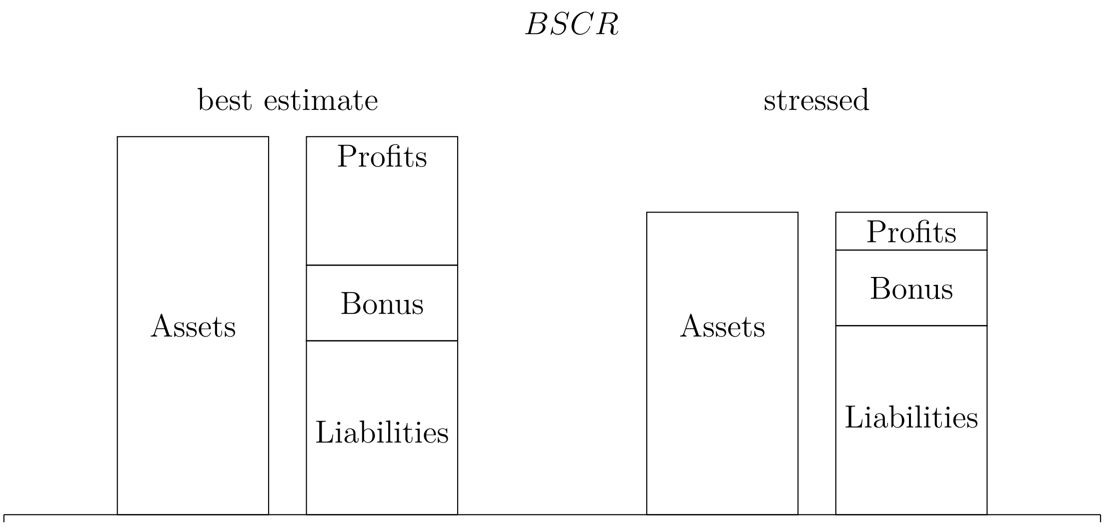


Figure 2: Risk absorbing effect of future profit sharing I

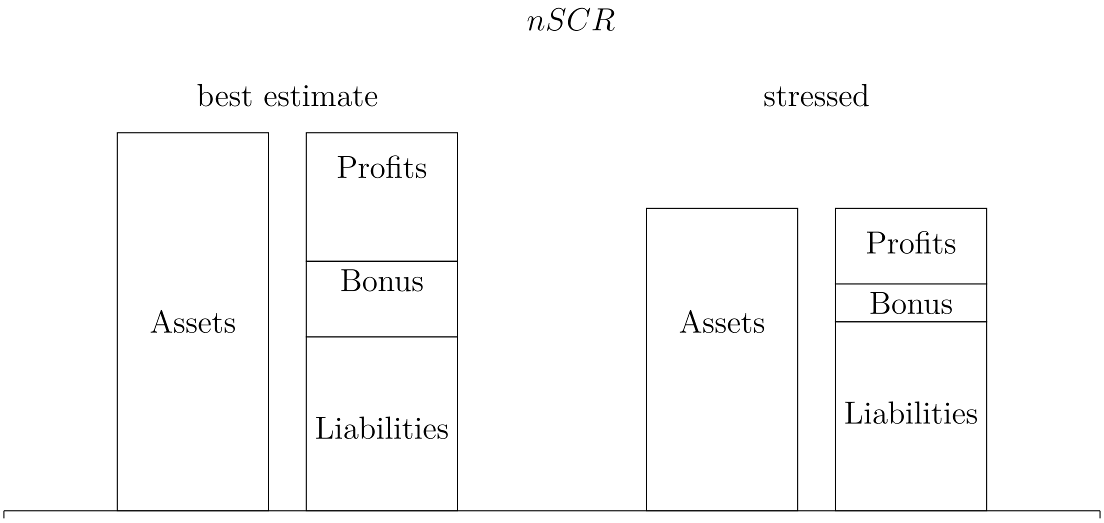


Figure 3: Risk absorbing effect of future profit sharing II

3 Product design and parameter assumptions

3.1 Product design

3.1.1 Premiums

In this paper two forms of the product are considered: single premium contracts and regular premium contracts. With a single premium contract, the policyholder has to pay only a lump-sum at the beginning of the contract period. Concluding a regular premium contract, the policyholder commits to pay a premium at the beginning of every month until the end of the contract period, death of the policyholder or lapse of the policy. The premium income is immediately used to buy shares of the investment fund after deduction of acquisition charges. Let T denote the policy term in years, then $t = 0, \ldots, 12 \cdot T$ is counting the time steps (months). A premium payment at time t is denoted by P_t.

3.1.2 Charges

Three kinds of expenses can be identified regarding a standard unit-linked insurance product: acquisition expenses, fixed monthly expenses, and variable monthly expenses. In order to refinance, the insurer deducts charges from the investment fund. These charges represent the prudent projected expenses[13]. The prudent projected expenses consist of the expected expenses plus a risk margin. The charges at time t are denoted as $acharges_t$, $fcharges_t$ and $vcharges_t$. The acquisition charges for regular premium policies are calculated with expected interest rates but without any mortality or lapse assumptions. The fixed monthly charges ($fcharges_t$) are considered to be deterministic and constant for all t, while the variable charges ($vcharges_t$) are driven by the current investment fund value[14]. The acquisition charges are immediately deducted from the premiums; in the single premium case, they are deducted from the single premium at once, in the regular premium case, the acquisition charges are decomposed in small payments and deducted from the premiums (for maximum five years). The incurred monthly (fixed and variable) expenses are paid at the end of every month. In order to finance the incurred expenses, the insurer withdraws an amount equal to the prudent projected expenses from the investment fund at the beginning of every month and deposits it on a bank account earning the risk-free interest rate. Let FV_t denote the investment fund value at time t, P^{tot} denote the

[13]See 3.3 for assumptions of expenses the insurer expects to experience.

[14]See 3.5 for more details to the calculation of charges.

single premium (in the single premium policy case) or the total amount of regular premiums (in the regular premium policy case, $P^{tot} = 12 \cdot P_t \cdot T$) and let $acharges^{rate}$ and $vcharges^{rate}$ denote the rates of the charges, then the following equations hold[15]:

$$acharges_t = P^{tot} \cdot acharges^{rate} \quad \text{for} \quad t = 0 \quad \text{single premium} \tag{36}$$

$$acharges_t = \frac{P^{tot} \cdot acharges^{rate}}{\ddot{a}_{\overline{s}|er}} \quad \text{for} \quad t \in [0, s-1] \quad \text{regular premium} \tag{37}$$

$$vcharges_t = vcharges^{rate} \cdot FV_t \tag{38}$$

3.1.3 Mortality

German DAV 2008 T mortality tables are used for prudent mortality assumptions[16]. Uniform distribution of deaths is used as an assumption for fractional ages. For integer x and $t \in [0, 1]$ the probability of a x-year-old to die in the ongoing year is uniformly distributed over the year, therefore

$$_tq_x = tq_x \tag{39}$$

Let NP_t denote the number of policies (which is equal to the number of policyholders) at time t, then the number of policies at time $t+1$ without considering lapses is:

$$NP_{t+1} = NP_t - NP_{\lfloor \frac{t}{12} \rfloor \cdot 12} \cdot \frac{1}{12} q_x \tag{40}$$

3.1.4 Death benefits

Death benefits are paid at the end of the month. Four kinds of policies are considered which include different guaranteed death benefits. The guarantees refer to the current investment fund and/or the premiums. Then, the death benefits DB_t at time t are defined as:

policy A: $DB_t = \max(1.1 \cdot FV_t, P^{tot})$

policy B: $DB_t = \max(FV_t, 0.5 \cdot P^{tot})$

policy C: $DB_t = 1.1 \cdot FV_t$

[15] With $s = \max\{12 \cdot P^{tot}, 60\}$ and an expected risk-free interest rate er (see 4.1 for more information).

[16] See Appendix B.

policy D: $DB_t = FV_t + 0.1 \cdot P^{tot}$

Like the charges, death benefits are also financed by withdrawing an amount from the investment fund at the beginning of the month. The amount withdrawn from the investment fund is referred to as the "risk premium" and denotes the prudent estimated excess of the death benefits over the investment fund value. Let q_x be the probability of a x-year-old dying the ongoing year and let RP_t denote the risk premium at time t, then the following equation holds:

$$RP_t = (DB_t - FV_t)\,\frac{q_x}{12 - q_x} \tag{41}$$

The risk premium is withdrawn from the investment funds at the beginning of the month and deposited on a bank account earning the risk free interest rate. Therefore, in case the death benefit paid to the policyholder is larger than the value of the amount of shares of the investment funds associated with the policy, the risk premium is used to close the gap. It is worth to notice that the risk premium is calculated at the beginning of the month with respect to the investment funds value at the beginning of the month while the death benefit is calculated with respect to the investment funds value at the end of the month. An unfavorable development of the investment funds during the month can lead to insufficient funds and therefore to negative profits for the insurer.

3.2 Lapses

Evaluating the value of the policies in its portfolio, the insurance company must take into account, that the insured might use his option to surrender, withdraw, or lapse his policy[17]. There are several factors that influence the number of lapses: the remaining policy term, the performance of the policy compared to other products, the age of the policyholder, unemployment rates, growth of the GDP, the rating of the insurance company, marketing and marketing channels as well as personal reasons[18]. Lapses triggered by these factors are not incorporated in this model in particular but combined and defined as irrational lapse and modeled by deterministic lapse rates.
Rational lapse is triggered by the value of the policy to the policyholder, more precisely, the surrender value of the policy. Rational lapse also often is referred to as dynamic policyholder behavior, since it cannot be modeled with deterministic assumptions. In literature, rational lapse is often used in connection with the valuation of a surrender option and therefore lapses are assumed to occur at any time the surrender value is larger than the value of the policy. Note that this definition of rational lapses differs from the rational lapse as presented in this paper. Dynamic policyholder behavior should be carefully managed by the insurance company because changes might be excessive and lead to huge financial losses. This paper also examines rational lapses (dynamic lapse functions)[19][20].

3.2.1 Irrational lapse

The irrational lapse is assumed to evolve with a deterministic monotonically decreasing lapse rate lr_t^{det}. It is useful to work with a annual lapse rate at first: Let alr_s^{det} denote the annual lapse rate with $s = 1, \dots, T$ and alr^α, alr^β and alr^γ denote a start, multiplier and floor value, then the annual lapse rate is defined as

$$alr_s^{det} = \max\{alr^\alpha - alr^\beta s, alr^\gamma\} \tag{42}$$

and the conversion equation is:

$$1 - alr^{det}_{\lfloor \frac{t}{12}+1 \rfloor} = \left(1 - lr_t^{det}\right)^{12} \tag{43}$$

[17]The three expressions are equally used in literature.

[18]For more information about lapse, see Anzilli & De Cesare (2007), Cerchiara *et al.* (2008), Kuo *et al.* (2003), Mauer & Holden (2007), Bacinello (2003), Cox & Lin (2006), Outreville (1990) and Prestele (2006).

[19]See chapter 7.

[20]See CEIOPS (2009a) for more information about irrational and rational lapses.

The number of policies at time $t+1$ is then without regarding mortality or rational lapses:

$$NP_{t+1} = NP_t \left(1 - lr_t^{det}\right) \tag{44}$$

3.2.2 Lapse fees

The insurance company is allowed to deduct a lapse fee from the surrender value of the policy in most European countries[21]. The main reasons for lapse fees are adverse selection, administration expenses, acquisition expenses and solvency[22].

Lapse fees are set to have a deterministic and monotonically decreasing lapse fee rate. In case a policyholder decides to surrender his policy, he receives the investment fund value less the lapse fee. Let lf_t^{rate} denote the lapse fee rate and lf^{α}, lf^{β} and lf^{γ} denote a start, multiplier and floor value, then the lapse fee rate is defined as

$$lf_t^{rate} = \max\{lf^{\alpha} - lf^{\beta} \left\lfloor \frac{t}{12} \right\rfloor, lf^{\gamma}\} \tag{45}$$

and the surrender value SV_t is defined as:

$$SV_t = FV_t \left(1 - lf_t^{rate}\right) \tag{46}$$

3.3 Best estimate assumptions and expenses

Best estimate assumptions are formulated for mortality, seperate assumptions are used for expenses.

- The best estimate mortality is assumed to be a constant fraction of the prudent mortality:

$$q_x' = 0.6 q_x \tag{47}$$

- The fixed monthly expenses are considered to be deterministic but monthly increasing with a expeses inflation factor. Let $cinf$ denote a constant expenses inflation rate (per annum) and cpu' (expenses per unit) denote the constant fixed expenses of one policy, then the fixed monthly expenses of one policy at time t can be expressed by

$$fexpenses_t' = cpu' \left(1 + cinf\right)^{\frac{t}{12}} \tag{48}$$

[21] E.g. in Germany (see VVG §169(5)), but not in France (see Helfenstein & Barnshaw (2003)), Norway (see Nordahl (2008)).

[22] See DAV-Arbeitsgruppe Stornoabzüge (2007) and Gatzert (2009) for further information.

- The variable expenses are assumed to be zero.
- The aquisition charges equal the aquisition expenses.

3.4 Bonus system

The insurer uses prudent and best estimate assumptions for mortality and different assumptions for charges and expenses because of prudence. Therefore, in the long run, the insurer will make profits out of the assumption of parameters. According to German law, these profits have to be shared with the policyholders. Two kinds of profits can be identified: mortality profits are profits generated by mortality risk taking and expense profits are profits generated by expenses risk taking. Expense profits also include profits from lapse fees and kickbacks[23]. Profits are generated every month and deposited on a bank account earning the risk free interest rate. At the end of the year the insurer credits at least 75% of the mortality profits and at least 50% of the expense profits to the policyholders investment fund. The rest of the profits are profits of the insurance company and denote the value of the policy (discounted at time $t = 0$) to the insurer.

[23]The investment fund management pays kickbacks to the insurer. Kickbacks are seen as an allowance on management fees due to a high transaction volume.

3.5 Parameter assumptions

The parameters set in this section represent the standard setting and are used unless otherwise noted.

Parameter	Value	Description	Category
T	30	policy term in years	general
$gender$	male	gender of the policyholders	
x	30	age of the policyholders at $t = 0$	
NP_0	10000	number of policyholders at $t = 0$	
P_0	100000	single premium in Euro	
$P_{t=0,\dots,12\cdot T}$	305	regular premium in Euro	
$acharges^{rate}$	6%	acquisition charges in per cent of P^{tot}	charges
cpu	4	fixed charges per policy per month in Euro	
$vcharges^{rate}$	0%	variable charges per month in per cent of FV_t (single premium case)	
$vcharges^{rate}$	0.15%	variable charges per month in per cent of FV_t (regular premium case)	
$aexpenses^{rate\prime}$	6%	acquisition expenses in per cent of P^{tot}	expenses
cpu'	4	fixed expenses per policy per month in Euro	
$cinf$	2%	fixed expenses inflation per annum	
$vexpenses^{rate\prime}$	0%	variable expenses per month in per cent of FV_t	
alr^{α}	12%	start value of the alr_s^{det} function	lapse
alr^{β}	2%	multiplier value of the alr_s^{det} function	
alr^{γ}	2%	floor value of the alr_s^{det} function	
lf^{α}	5%	start value of the lf_t^{rate} function	
lf^{β}	0.5%	multiplier value of the lf_t^{rate} function	
lf^{γ}	0%	floor value of the lf_t^{rate} function	
rb^{rate}	75%	risk profit participation rate	bonus system
cb^{rate}	50%	expense profit participation rate	

Table 2: Parameter assumptions

4 Simulations

4.1 Financial market model

The financial market model consists of one risky asset (e.g. a share) and a riskfree investment possibility (e.g. a state bond). The risky asset is modeled by using the standard Black-Scholes-Merton model, while the interest rates are modeled with the Cox-Ingersoll-Ross model.

4.1.1 Investment fund

The investment fund contains only risky assets but is modeled with respect to investment fund fees and kickbacks to the insurance company.
Let S_t denote the value of one share of the risky asset with a constant volatility σ, r_t the risk-free short-rate[24] and W_t a Brownian motion at time $t \in [0, T]$, then S_t fulfills the following sde in a risk-free world[25]:

$$dS_t = r_t S_t dt + \sigma S_t dW_t \tag{49}$$

This sde has a known solution:

$$\begin{align} S_t &= S_{t-1}\ exp\left(\int_{t-1}^{t}\left(r_s - \frac{\sigma^2}{2}\right)ds + \int_{t-1}^{t}\sigma\ dW_s\right) \tag{50} \\ &= S_{t-1}\ exp\left(\int_{t-1}^{t} r_s ds - \frac{\sigma^2}{2} + \sigma\epsilon\right) \qquad \text{with } \epsilon \sim \mathcal{N}(0,1) \tag{51} \end{align}$$

Now let $FundFee$ denote a constant rate of fees, which will be retained by the investment fund management and let A_t denote the value of one share of the investment fund, then

$$dA_t = r_t A_t dt + \sigma A_t dW_t + \ln\left(1 - FundFee\right) A_t dt \tag{52}$$

describes the movements of the investment fund. The investment fund is modeled like a dividend paying share[26]. The solution of this sde can be

[24]Assuming an adapted interest rate process r_t Shreve (2000, page 215).
[25]See Shreve (2000, page 214-217).
[26]See Shreve (2000, page 234-240).

written as

$$A_t = A_{t-1}\ exp\left(\int_{t-1}^{t}\left(r_s - \frac{\sigma^2}{2}\ + \ln\left(1 - FundFee\right)\right)ds + \int_{t-1}^{t}\sigma\ dW_s\right) \tag{53}$$

$$= A_{t-1}\ exp\left(\int_{t-1}^{t} r_s ds - \frac{\sigma^2}{2} + \sigma\epsilon\right)(1 - FundFee) \qquad \text{with } \epsilon \sim \mathcal{N}(0,1) \tag{54}$$

$$= A_{t-1}\ \frac{S_t}{S_{t-1}}\left(1 - FundFee\right) \tag{55}$$

The amount of fees retained by the investment fund management per investment fund share at time t is then

$$FG_t = A_{t-1}\ \frac{S_t}{S_{t-1}} FundFee \tag{56}$$

Since the investment fund fee is usually given in an annual form, denoted by $aFundFee$, the following conversion formula is used:

$$1 - aFundFee = \left(1 - FundFee\right)^{12} \tag{57}$$

The kickbacks are paid by the investment fund management to the insurer and are financed with the investment fund managment fees[27]. The amount of kickbacks per investment fund share is

$$\text{Kickbacks (per share)} = A_t \cdot \text{kickbackrate} \tag{58}$$

4.1.2 Interest rates

The Cox-Ingersoll-Ross model is used to model the short rate[28]. Unfortunately, this model has no closed-form solution, but the interest rates are always positive. Let lm denote the constant long run short rate, mrs the constant mean reversion speed, σ_r the volatility of the interest rates and W_t^r a Brownian motion, then the model for the short rate process r_t is

$$dr_t = mrs\left(lm - r_t\right)dt + \sigma_r\sqrt{r_t}dW_t^r \tag{59}$$

It is useful to introduce some additional expressions for the interest rates[29]. Let $fr_{[t-1,t],i}$ denote the forward rate for $t \in [t-1, t]$ for the simulation path

[27] Therefore, the rate of kickbacks should be chosen smaller than the rate of investment fund management fees.

[28] See Shreve (2000, page 151-153).

[29] See chapter 5.

i; the zero coupon bond swap rate at maturity t is denoted by $zcsr_{t,i}$. Then the following equations hold[30]:

$$1 + fr_{[t-1,t],i} = \exp\left(\int_{t-1}^{t} r_{s,i} ds\right) \tag{60}$$

$$(1 + zcsr_{t,i})^t = (1 + zscr_{t-1,i})^{t-1}\left(1 + fr_{[t-1,t],i}\right) \tag{61}$$

4.1.3 Implementation

To implement the financial market model, the first step is the simulation of the two Brownian motions W_t and W_t^r. Sometimes, it is useful to correlate the stochastic parts of the risky and the risk-free asset. Since the interest rate already influences the risky asset by design of the process (the drift of the geometric Brownian motion) and with no influence of the risky asset on the interest rate in this model yet, it might be useful to define the Brownian motions as follows:

$$W_t = B_t \tag{62}$$

$$W_t^r = \rho W_t + \sqrt{1-\rho^2} B_t^r \tag{63}$$

with two independent standard Brownian motions B_t and B_t^r and correlation factor ρ.

Unit-linked products are usually calculated every month, therefore the simulation steps have a time span of one month. With the policy term T and n simulation runs, $12 \cdot T \cdot 2 \cdot n$ independent standard normal random variables must be generated[31]. Note that all variables are defined pathwise in this section, the index i has been left out for reason of readability[32].

Let X_t and X_t^r be independent standard normal random variables with $t = 0, \ldots, 12 \cdot T$, then the random variables for the simulations can be obtained with:

$$\epsilon_t = X_t^1 \tag{64}$$

$$\epsilon_t^r = \rho \epsilon_t + \sqrt{1-\rho^2} X_t^r \tag{65}$$

The second step is the simulation of the interest rates. Since there is no closed-form solution, a discretization is used to obtain the short rates. The

[30]See Daniel & Vaaler (2007) for more information about interest rates.

[31]The random number generator included in the Microsoft Office 2003 was used for the simulations.

[32]Therefore, i.e. $S_t = S_{t,i}$ or $r_t = r_{t,i}$

well known Euler-Maruyama discretization scheme has been used because of its simplicity[33]. Let r_0 be the starting value, then the following short rates for $t = 1, \dots, 12 \cdot T$ are defined by:

$$r_t = r_{t-1} + \frac{1}{12} mrs \left(lm - r_{t-1}\right) + \sigma_r \sqrt{\frac{1}{12} r_{t-1}}\ \epsilon_t^r \tag{66}$$

The additional expressions for the interest rates can easily be obtained by

$$1 + fr_{[t-1,t]} = \exp\left(\frac{1}{12} r_{t-1}\right) \tag{67}$$

$$\left(1 + zcsr_t\right)^t = \left(1 + zscr_{t-1}\right)^{t-1} \left(1 + fr_{[t-1,t]}\right) \tag{68}$$

The zero coupon bond swap rates at maturity t are monthly; it is useful to annualize them[34]. The annualized zero coupon bond swap rates at maturity t, $azcsr_t$, are defined by

$$azcsr_t = \left(1 + zcsr_t\right)^{12} - 1 \tag{69}$$

For some calculations, the insurer needs a projected interest rate (e.g. acquisition charges in the regular premiums case). In order to avoid nested simulations, the expected value, which is calculated with the discretization of the short rate formula is used. Let er_t denote the expected short rate and let $er_0 = r_0$ be a starting value, then the following equation holds:

$$er_t = er_{t-1} + \frac{1}{12} mrs \left(lm - er_{t-1}\right) \tag{70}$$

The third step, the simulation of the risky asset and the investment fund, is straight forward. With the starting value $S_0 = A_0$ and using the forward rate, the values of one share of the risky asset S_t and the values of one share of the investment fund A_t for $t = 1, \dots, 12 \cdot T$ can be obtained by

$$S_t = S_{t-1}\ exp\left(\ln\left(1 + fr_{[t-1,t]}\right) - \frac{1}{12}\frac{\sigma^2}{2} + \sigma\sqrt{\frac{1}{12}}\epsilon_t\right) \tag{71}$$

$$A_t = A_{t-1}\ \frac{S_t}{S_{t-1}} \left(1 - FundFee\right) \tag{72}$$

[33]See Alfonsi (2006, page 3).

[34]The QIS4 interest rates stress scenario is performed on annual zero coupon bond swap rates.

4.2 Parameter assumptions

The parameters set in this section represent the standard setting and are used unless otherwise noted.

Parameter	Value	Description	Category
n	5000	number of simulations	general
ρ	0	correlation between the Brownian motions	
S_0	100	starting value of the risky asset	risky asset
σ	20%	volatility per annum	
r_0	4%	starting value	interest rate
mrs	0.3	mean reversion speed	
lm	4.5%	long run short rate	
σ_r	2.5%	volatility per annum	
$aFundFee$	1.5%	investment fund fee per annum	investment fund
$akickbackrate$	0.5%	kickback rate per annum	

Table 3: Financial market model parameter assumptions

4.3 Simulation steps

The following figure describes the simulation sequence of one simulation path of the liability portfolio projection component of the Excel-tool:

For $t = 0$:

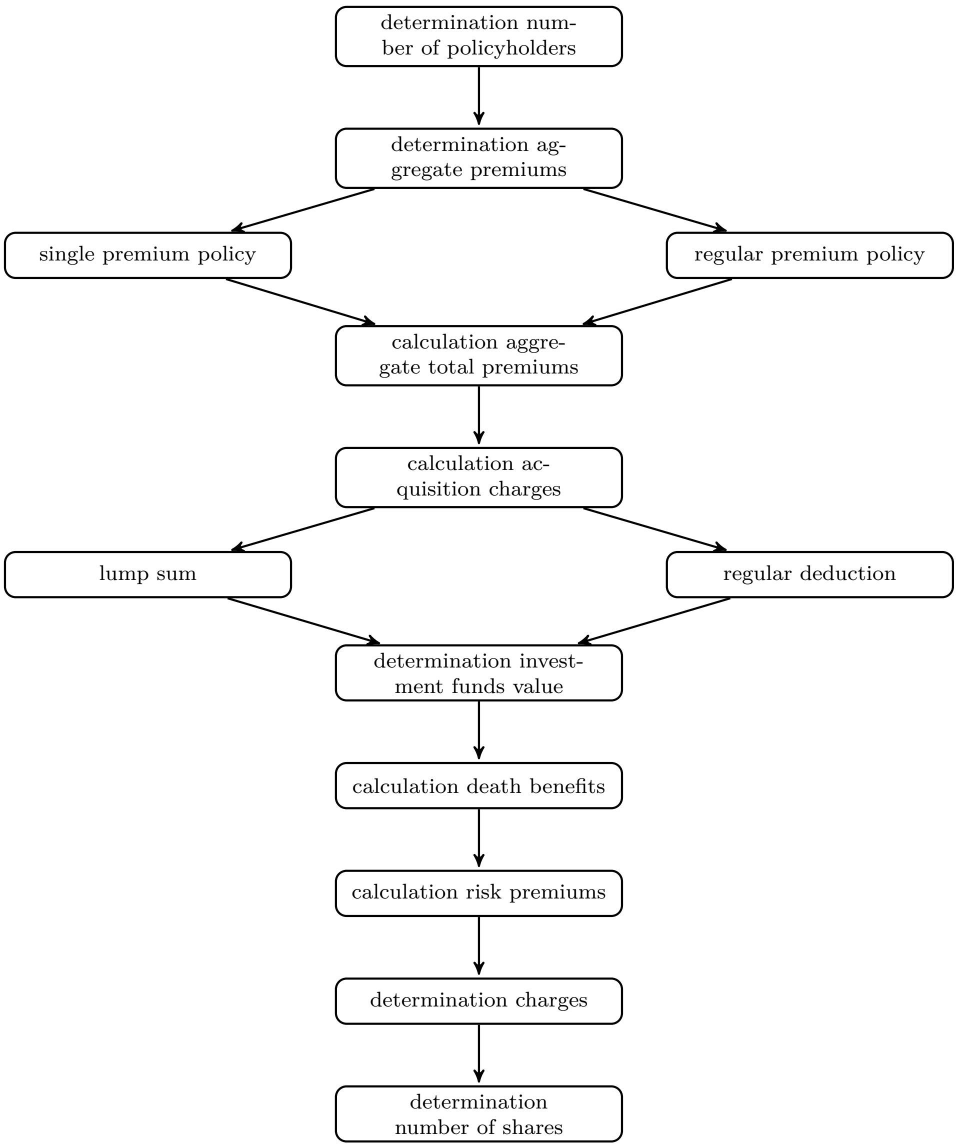


Figure 4: Simulation steps I

For $t = 1, \ldots, 12T$:

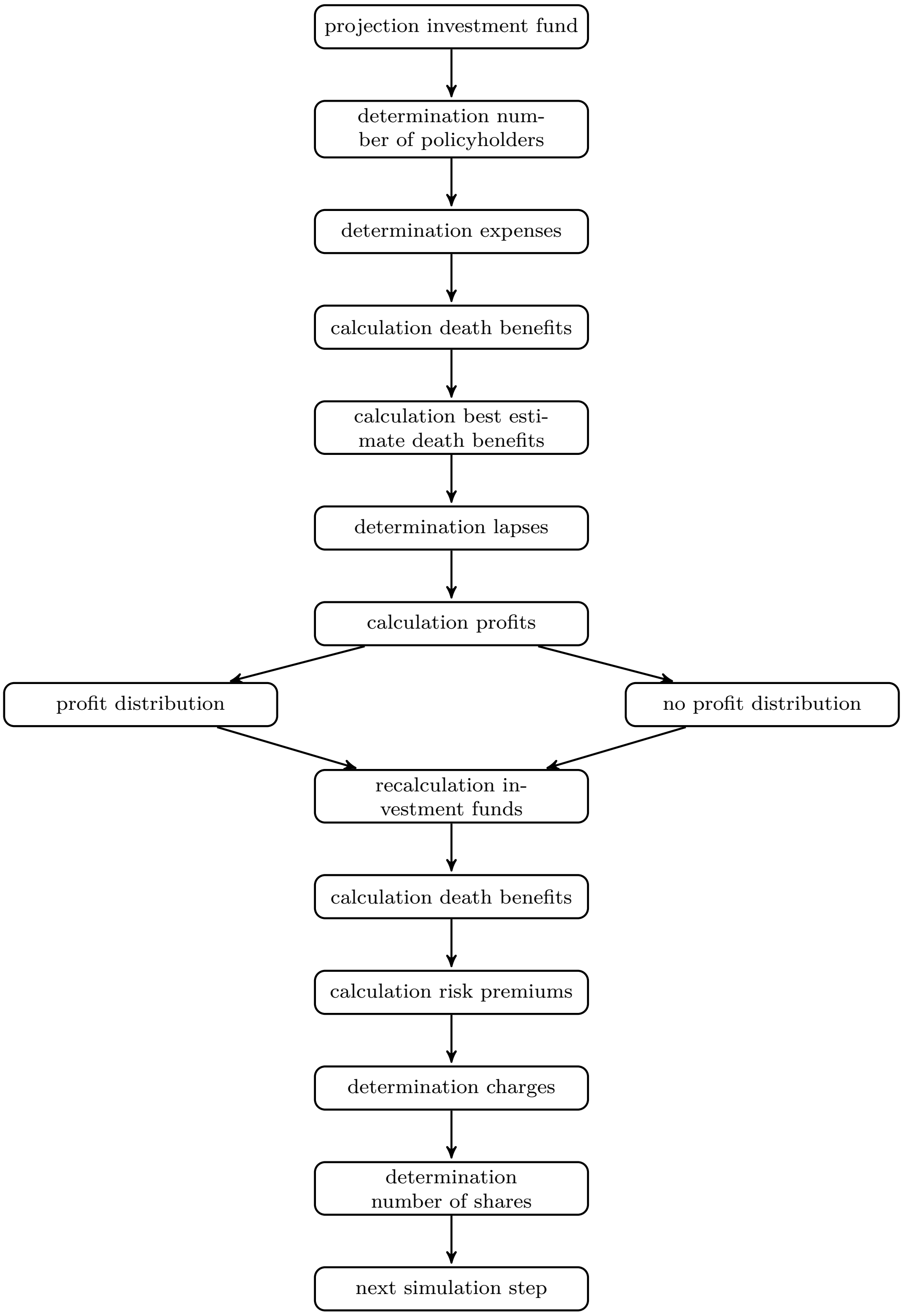


Figure 5: Simulation steps II

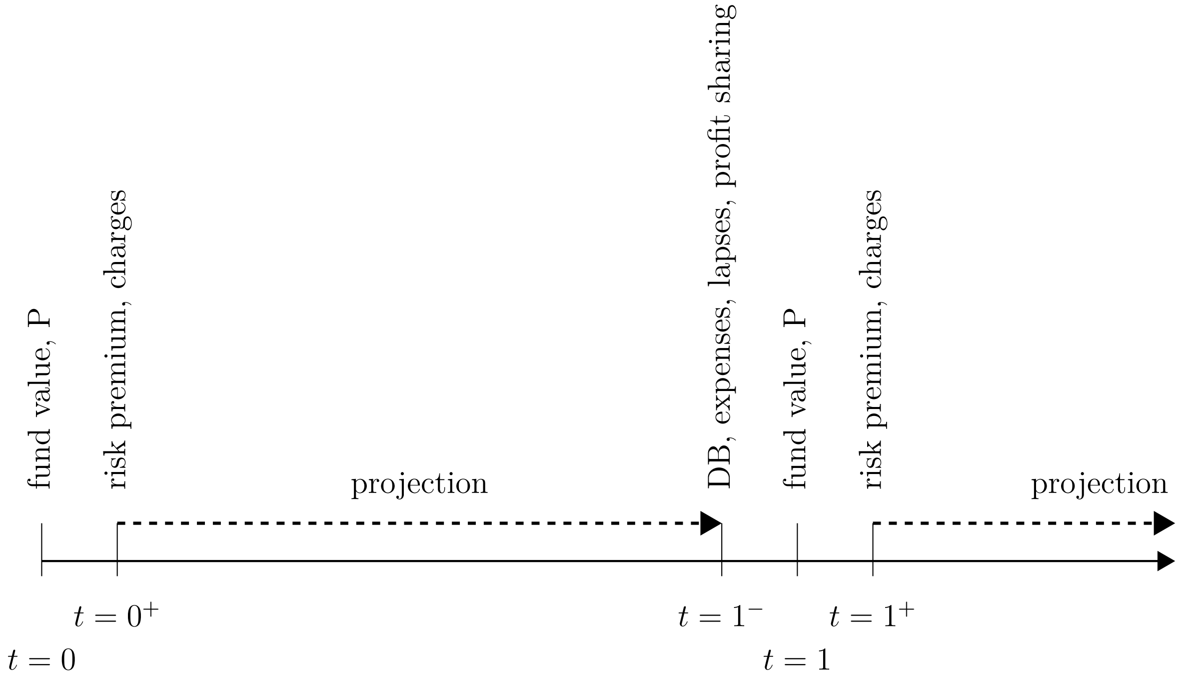


Figure 6: Timeline

4.4 Numerical results

Policy type	P type	SCR	Profits	Solvency ratio	SCR ratio	SCR ratio – Solvency I
A	sin	€ 15,03 mln	€ 40,10 mln	266,77%	1,60%	1,00%
B	sin	€ 15,01 mln	€ 39,63 mln	264,09%	1,60%	1,00%
C	sin	€ 15,17 mln	€ 39,87 mln	262,80%	1,61%	1,00%
D	sin	€ 15,07 mln	€ 39,75 mln	263,87%	1,60%	1,00%
A	reg	€ 14,76 mln	€ 40,15 mln	272,02%	1,43%	0,00%
B	reg	€ 14,70 mln	€ 39,98 mln	272,02%	1,42%	0,00%
C	reg	€ 14,75 mln	€ 40,06 mln	271,55%	1,43%	0,00%
D	reg	€ 14,75 mln	€ 40,05 mln	271,57%	1,43%	0,00%

Table 4: Numerical results I

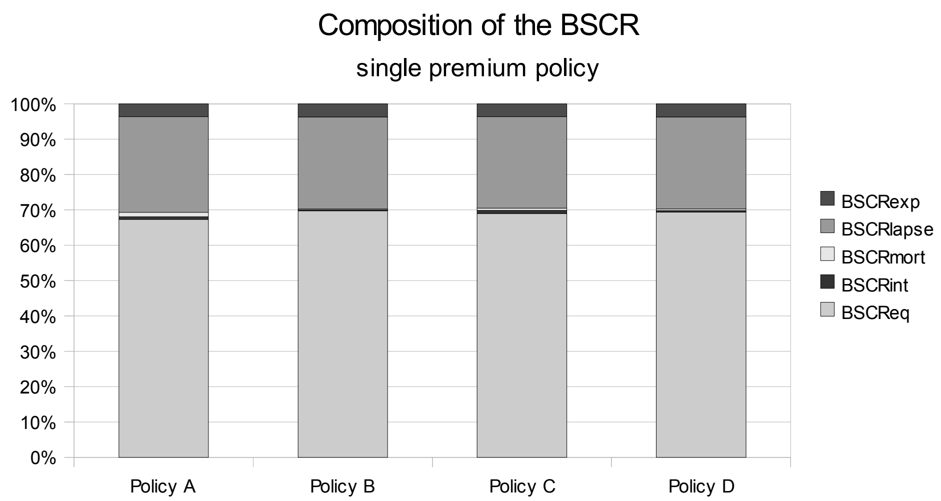


Figure 7: Composition of the BSCR – single premium

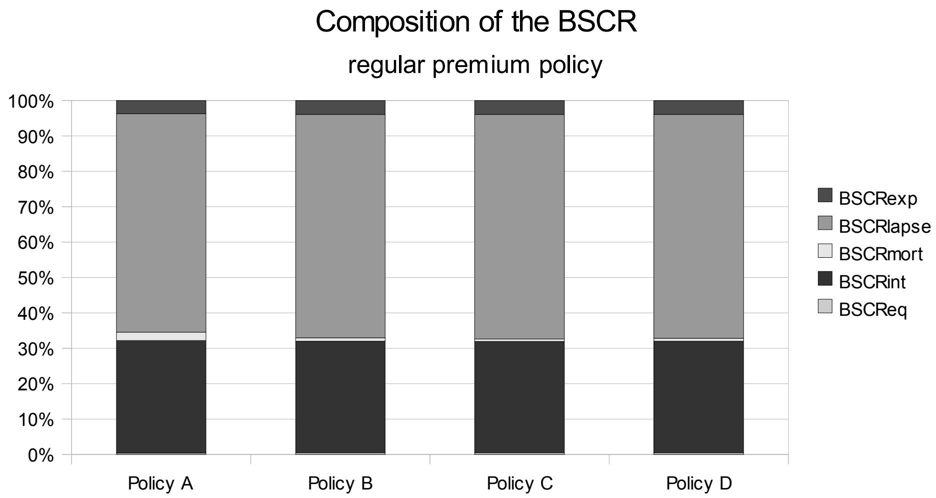


Figure 8: Composition of the BSCR – regular premium

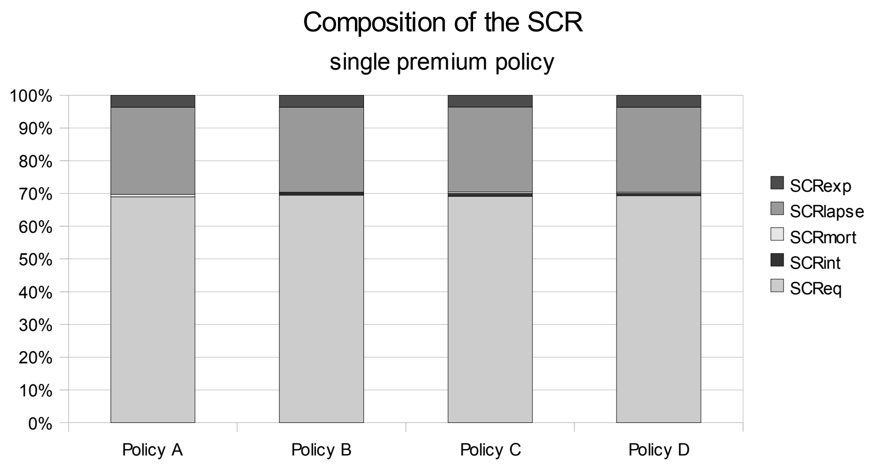


Figure 9: Composition of the nSCR – single premium

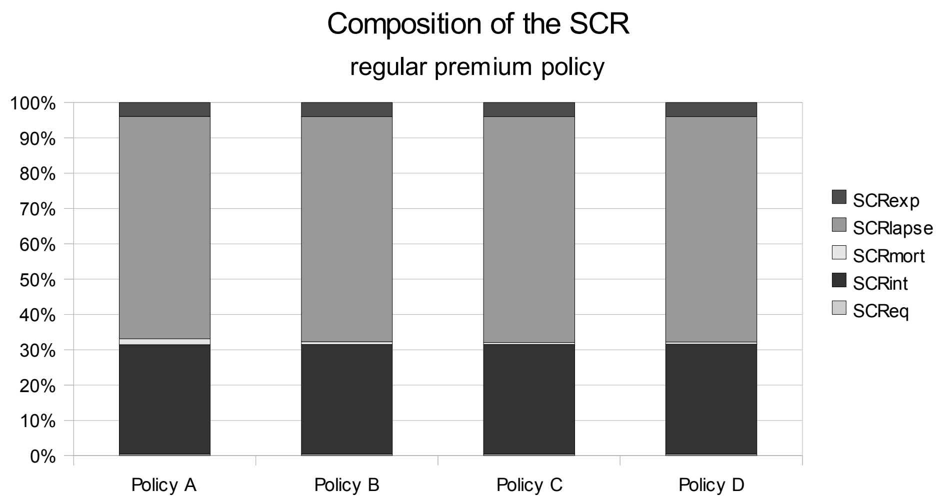


Figure 10: Composition of the nSCR – regular premium

Table 4 presents the simulated SCR's and the insurer's profits for all four kinds of policies with both premium types (”sin” for single premium and ”reg” for regular premium). Furthermore, it presents two important financial ratios: Solvency ratio and SCR ratio[35]. Figures 7 to 10 show the compositions of the BCSR and the nSCR before diversification.

The first observation is that market risks and lapse risk dominate the risk structure of the respective product. Throughout all simulation runs the long-term increase of the lapse rates proved to be the relevant stress scenario. Expense risk and mortality risk are both almost negligible. Therefore, the type of death benefits has also only little impact on the solvency capital requirement.

Secondly, comparing the results of the simulations, the premium type of the policy proves to be very important for the policies' risk structure. Although the regular premium policy is just insignificantly more risky than the single premium policy (by comparing the SCR ratio), interest rate risk represents market risks almost completely. This fact is not surprising, since, with a regular premium policy, the fund value is small at the beginning. On the opposite, the market risk of a single premium policy is dominated by the equity risk. A shock of interest rates does not have a significant impact on the profits. Since a change of interest rates does affect the discounting of future profits as well as the trend of the risky assets and since the profits

[35]With Solvency ratio = $\frac{\Pi}{SCR}$ and SCR ratio = $\frac{SCR}{P^{tot}}$.

are mostly generated or triggered by the investment fund value, both effects seem to offset each other.

Thirdly, the solvency capital requirement calculated with the standard formula of the Solvency II framework that does not even include operational risk yet, seems to be much higher than the solvency capital requirement calculated according to the Solvency I framework (by a factor of 1.6 or higher, comparing the SCR ratios). For the regular premium policy, Solvency I requires only little solvency capital at the beginning of the policy term and the biggest amount of solvency capital at the end of the policy term although this is illogical since the risk obviously decreases to the end of the policy term in general.

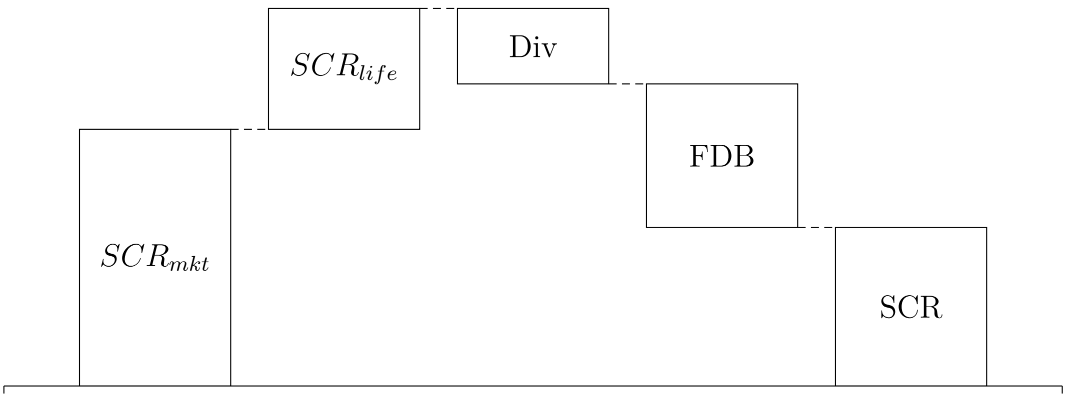


Figure 11: SCR - Structure - single premium

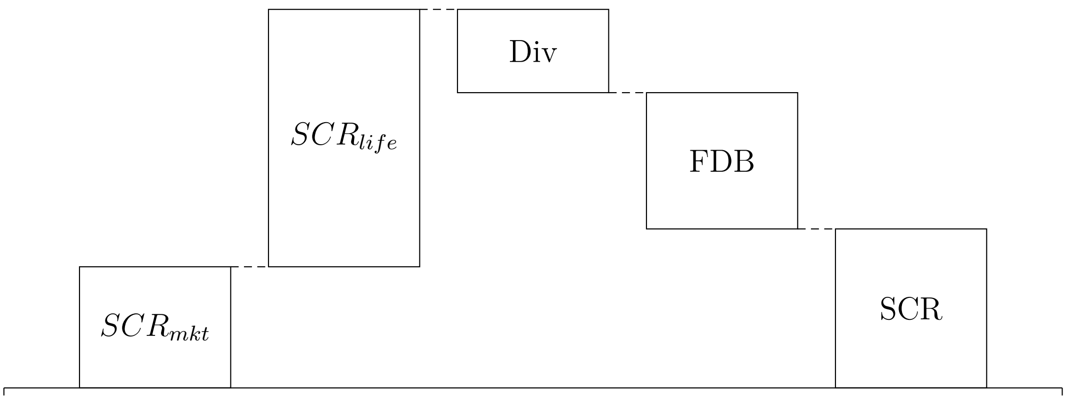


Figure 12: SCR - Structure - regular premium

Profits distribution	with profit sharing	Policy type A	P type sin		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
Profits	€ 40,10 mln	€ 0,60 mln	€ 39,50 mln	€ 7,41 mln	€ 32,62 mln
Profits after mort-shock	€ 39,97 mln	€ 0,51 mln	€ 39,46 mln	€ 7,41 mln	€ 32,58 mln
Profits after exp-shock	€ 39,42 mln	€ 0,60 mln	€ 38,82 mln	€ 7,41 mln	€ 32,61 mln
Profits after lapse-shock	€ 35,16 mln	€ 0,41 mln	€ 34,75 mln	€ 10,09 mln	€ 25,03 mln
Profits after eq-shock	€ 27,31 mln	€ 0,73 mln	€ 26,58 mln	€ 5,03 mln	€ 22,08 mln
Profits after int-shock	€ 40,12 mln	€ 0,45 mln	€ 39,68 mln	€ 7,41 mln	€ 32,67 mln

Profits distribution	without profit sharing	Policy type A	P type sin		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
Profits	€ 78,82 mln	€ 2,42 mln	€ 76,40 mln	€ 14,69 mln	€ 62,77 mln
Profits after mort-shock	€ 78,38 mln	€ 2,05 mln	€ 76,33 mln	€ 14,69 mln	€ 62,69 mln
Profits after exp-shock	€ 77,50 mln	€ 2,42 mln	€ 75,08 mln	€ 14,69 mln	€ 62,77 mln
Profits after lapse-shock	€ 68,99 mln	€ 1,67 mln	€ 67,31 mln	€ 19,97 mln	€ 48,08 mln
Profits after eq-shock	€ 54,36 mln	€ 3,01 mln	€ 51,35 mln	€ 9,98 mln	€ 42,42 mln
Profits after int-shock	€ 78,53 mln	€ 1,77 mln	€ 76,76 mln	€ 14,69 mln	€ 62,86 mln

Profits distribution	with profit sharing	Policy type A	P type reg		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
Profits	€ 40,15 mln	€ 1,11 mln	€ 39,04 mln	€ 0,37 mln	€ 8,52 mln
Profits after mort-shock	€ 39,84 mln	€ 0,94 mln	€ 38,90 mln	€ 0,37 mln	€ 8,49 mln
Profits after exp-shock	€ 39,41 mln	€ 1,11 mln	€ 38,30 mln	€ 0,37 mln	€ 8,50 mln
Profits after lapse-shock	€ 28,35 mln	€ 0,84 mln	€ 27,51 mln	€ 0,47 mln	€ 5,95 mln
Profits after eq-shock	€ 40,06 mln	€ 1,11 mln	€ 38,95 mln	€ 0,37 mln	€ 8,50 mln
Profits after int-shock	€ 34,34 mln	€ 0,86 mln	€ 33,48 mln	€ 0,35 mln	€ 7,28 mln

Profits distribution	without profit sharing	Policy type A	P type reg		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
Profits	€ 74,81 mln	€ 4,68 mln	€ 70,12 mln	€ 0,72 mln	€ 15,31 mln
Profits after mort-shock	€ 73,97 mln	€ 3,98 mln	€ 69,99 mln	€ 0,72 mln	€ 15,28 mln
Profits after exp-shock	€ 73,48 mln	€ 4,68 mln	€ 68,80 mln	€ 0,72 mln	€ 15,31 mln
Profits after lapse-shock	€ 53,08 mln	€ 3,53 mln	€ 49,55 mln	€ 0,92 mln	€ 10,73 mln
Profits after eq-shock	€ 74,66 mln	€ 4,69 mln	€ 69,97 mln	€ 0,71 mln	€ 15,28 mln
Profits after int-shock	€ 63,63 mln	€ 3,59 mln	€ 60,04 mln	€ 0,68 mln	€ 13,07 mln

Table 5: Composition of the profits

Table 5 presents the impact of the stress-scenarios on the profits. Furthermore, the composition of the profits is shown. Note that profits from lapse fees and kickbacks are part of the profits from expenses. Table 5 also displays the impact of profit sharing on profits and its risk absorbing effect. Table 6 presents the effect of profit sharing in more detail. The insurer is able to mitigate the risk almost identical to the profit participation rates.

Policy type A	P type sin		
	with profit sharing	without profit sharing	risk mitigation
Profits	€ 40,10 mln	€ 78,82 mln	49%
SCR	€ 15,03 mln	€ 28,98 mln	48%
SCReq	€ 12,79 mln	€ 24,46 mln	48%
SCRint	€ 0,00 mln	€ 0,29 mln	100%
SCRmort	€ 0,13 mln	€ 0,44 mln	70%
SCRlapse	€ 4,94 mln	€ 9,83 mln	50%
SCRexp	€ 0,68 mln	€ 1,32 mln	49%

Policy type A	P type reg		
	with profit sharing	without profit sharing	risk mitigation
Profits	€ 40,15 mln	€ 74,81 mln	46%
SCR	€ 14,76 mln	€ 27,46 mln	46%
SCReq	€ 0,08 mln	€ 0,15 mln	43%
SCRint	€ 5,81 mln	€ 11,18 mln	48%
SCRmort	€ 0,30 mln	€ 0,84 mln	64%
SCRlapse	€ 11,80 mln	€ 21,73 mln	46%
SCRexp	€ 0,74 mln	€ 1,32 mln	44%

Table 6: Risk absorbing effect of future profit sharing

5 Stress scenarios

The solvency capital requirement is defined as the difference of the best estimate net asset value (profits) and the net asset value under stress[36]. The stress scenarios defined in this chapter originate from QIS4. The design of the regarded insurance product requires the consideration of the following risks: in the market risk module, the interest rate risk and the equity risk are relevant. Mortality risk, lapse risk and expense risk are the relevant risks from the life underwriting risk module. Note that CEIOPS proposes to adjust some of the stress scenarios of QIS4[37], these adjustments are not considered in this paper.

5.1 QIS4 stress scenarios

"interest rate risk" [38] – The interest rate risk module includes two stress scenarios: up-shift of the interest rate curve (zero coupon bond rate) and down-shift of the interest rate curve. The exact magnitude of the shifts can be found in the QIS4 tables[39].

"equity risk" [40] – The equity risk module contains an immediate loss of 32% of the risky assets[41].

"mortality risk" [42] – The mortality stress is defined as an increase of the mortality rates amounting to 10%.

"lapse risk" [43] – The lapse risk includes three stress scenarios: a long-term increase of the lapse rates, a long-term decrease of the lapse rates and a massive immediate lapse of 30% of the policyholders.

"expense risk" [44] – The expense risk stress scenario is defined as an increase of 10% in future expenses and an increased expenses inflation (+1% per annum).

[36] See chapter 2 for details.
[37] see the consultation papers for more information.
[38] See CEIOPS (2008c, pages 134-137).
[39] See appendix A.
[40] See CEIOPS (2008c, pages 137-143).
[41] The risky assets are assumed to belong to the asset category "Global".
[42] See CEIOPS (2008c, pages 162-164).
[43] See CEIOPS (2008c, pages 167-169).
[44] See CEIOPS (2008c, pages 169-170).

5.2 Implementation of the stress scenarios

The liability portfolio projection component of the Excel-tool is run through several loops during the calculation of the solvency capital requirement. The different stress scenarios shock the following parameters. Note that the simulated results of the financial market model are required, since the market stresses are performed on this results.

”interest rate risk” The stresses are applied to the annualized zero coupon bond swap rates at maturity t, $azcsr_t$. The relative changes of the interest rates due to an upward stress are denoted by s^{upward} and the relative changes due to a downward stress are denoted by $s^{downward}$. The stressed annualized zero coupon bond swap rates are then

$$azcsr_t^{upward} = azcsr_t \left(1 + s^{upward}\right) \tag{73}$$

and

$$azcsr_t^{downward} = azcsr_t \left(1 + s^{downward}\right) \tag{74}$$

The stressed zero coupon bond swap rates and forward rates are derived from the stressed annualized zero coupon bond swap rates using equations 69 and 68. The stressed interest rates are also used to derive the movements of the risky assets (as the drift of the geometric Brownian motion). Therefore, the interest rates stresses have an influence on the financial market model as a whole.

”equity risk” The equity stress is an immediate short term stress on the risky asset. Let $s_{equity}^{down} = -32\%$ denote the relative change of equity value, then

$$\left.\frac{S_1}{S_0}\right|_{stressed} = \frac{S_1}{S_0} \left(1 + s_{equity}^{down}\right) \tag{75}$$

The values of the risky asset are calculated with a recursive formula, therefore a change of S_1 and A_1 requires recalculation of S_t and A_t for all $t > 1$.

”mortality risk” The increase of 10% of the mortality rates applies to the best estimate mortality rates, therefore:

$$q_x^{\prime stressed} = 1.1 \cdot 0.6 q_x \tag{76}$$

"lapse risk" The relative changes are performed to the monthly deterministic lapse rates:

$$lr_t^{det/up} = 1.5 \cdot lr_t^{det} \qquad \forall t \tag{77}$$

$$lr_t^{det/down} = 0.5 \cdot lr_t^{det} \qquad \forall t \tag{78}$$

$$lr_t^{det/mass} = 1 - (1 - 0.3)^{\frac{1}{12}} \qquad \forall t \in [0, 11] \tag{79}$$

"expense risk" The increase of 10% of the expenses applies to the monthly fixed expenses and 1% is added to the expenses inflation rate, therefore:

$$cpu'^{stressed} = 1.1 cpu' \tag{80}$$

$$cinf^{stressed} = cinf + 0.01 \tag{81}$$

6 Linearities

The Solvency II standard formula is based on the assumption of linearity. Two types of linearity can be identified: Linearity within a risk and linearity between risks. Linearity within a risk ensures that the solvency capital requirement of a single risk module increases linearly with the risk factor. Following equation holds:

$$kSCR(X_i) = SCR(kX_i) \tag{82}$$

for any positive k and every risk i. The linearity between risks ensures that the separately calculated diversified solvency capital requirement of several risk modules equals the solvency capital requirement of a simultaneous shock with adjusted risk factors:

$$SCR^{k \cdot SES}(X) = \sqrt{\sum_{i,j} \rho_{i,j} SCR(kX_i) SCR(kX_j)} \tag{83}$$

with $X = \sum_i X_i$ and the single equivalent scenario SES[45].

Non-linearities can compromise the accuracy of the solvency capital requirement calculated with the standard formula. Excessive non-linearities nearing the defined stress scenarios can lead to significant changes of the solvency capital requirement. More crucial, non-linearities between risks can not be evaluated with the standard formula. It is possible that an insurance company facing unfavorable developments in several risk modules is in need of much more or much less capital than aggregated with the standard formula. Furthermore, the single equivalent scenario method requires both, linearity within a risk and linearity between risks[46].

Figures 13 and 14 show sensitivity graphs of the relevant risks, equity and lapse (up-shock) for a single premium policy (type A). For a regular premium policy (type A), sensitivity graphs of the interest rate (up-shock) and lapse (up-shock) are presented. The values on the x-axis denote the reduction factor for the risk from zero, denoting "no stress", to one, denoting "full QIS4 stress-scenario"[47]. The grey curves represent the impact on the profits while the black curves linear regression lines. All graphs indicate almost perfect linearity. Non-linearity can be found within the lapse risk. There is

[45] See chapter 8 for more information.

[46] See chapter 8.

[47] A value of 0.5 for the equity risk would denote an immediate loss of 16% of the risky assets.

also non-linearity between market risks and lapse risk as seen in figure 15. Here, the grey curves represent the impact on the profits of simultaneous stress-scenarios with adjusted risk factors, the black curves represent the total impact on the profits of seperately calculated stress-scenarios including diversification. This result is important for the single equivelent scenario[48].

[48]See chapter 8 for more information.

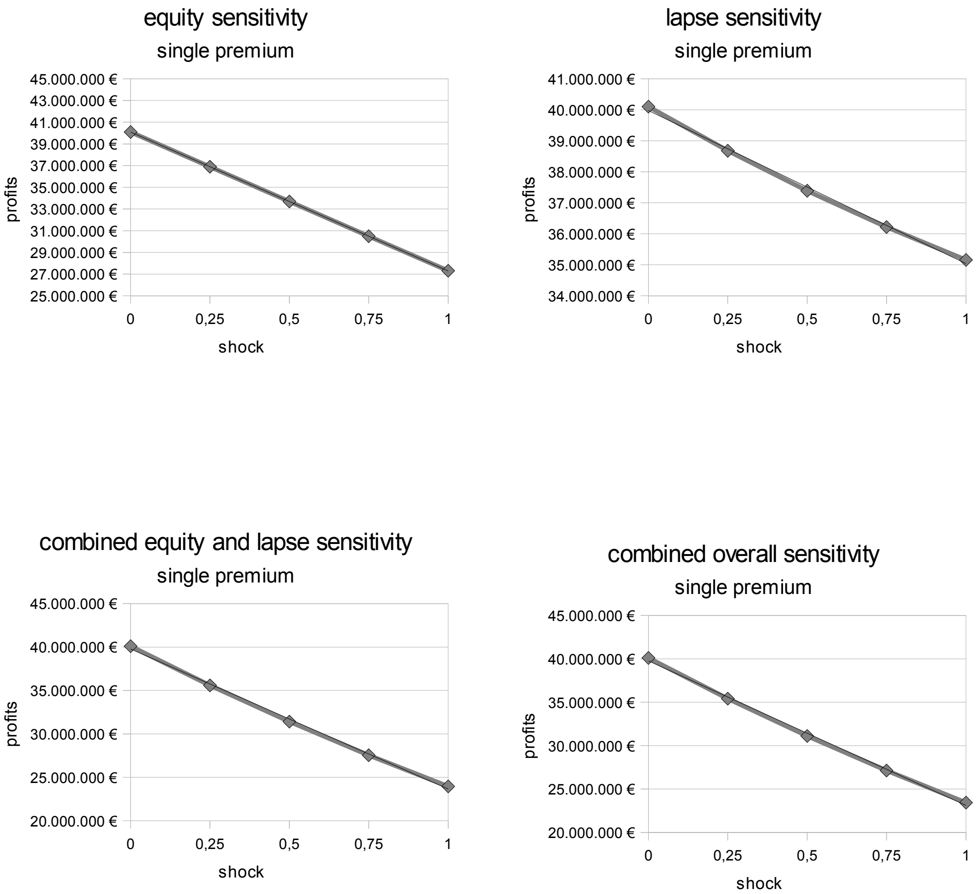


Figure 13: Numerical results - Linearities - single premium

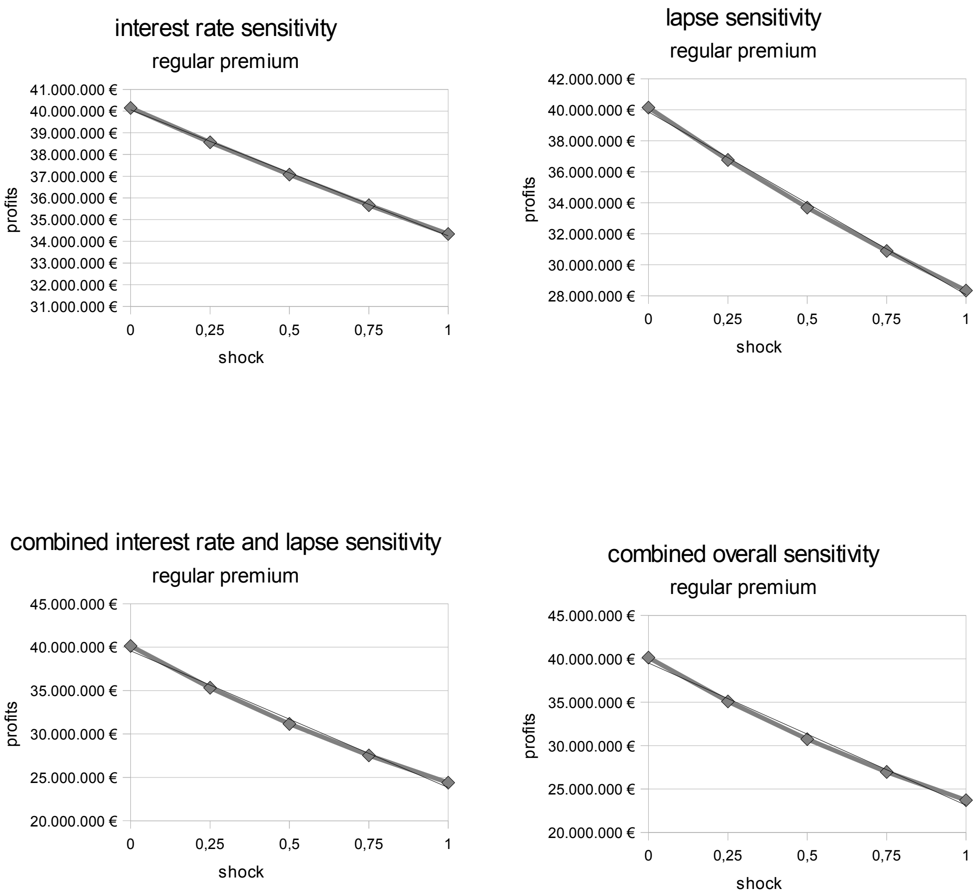
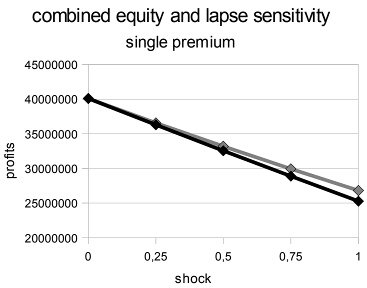


Figure 14: Numerical results - Linearities - regular premium



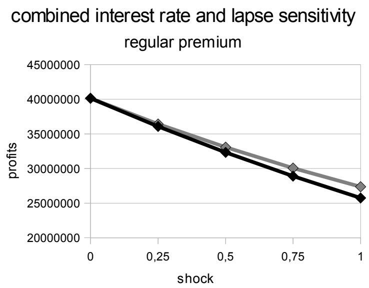


Figure 15: Numerical results - Non-linearities between risks

7 Dynamic policyholder behavior

Dynamic policyholder behavior is a major concern to actuaries. The lack of statistical data and the amount of factors that may influence the policyholder's behavior have to be taken into account and make it difficult to model or project the policyholder's actions. The challenge is even bigger considering a situation of a new product launch and therefore only little experience. On the other hand, it is common sense among actuaries that dynamic policyholder behavior, especially dynamic lapses, can be a major risk. Throughout the literature, there are indicators that suggest a more distinct dynamic behavior for unit-linked products[49] caused by a higher volatility in the ”value” of options, guarantees or the fund value. CEIOPS addresses the existence of options and guarantees as well as the financial markets as reasons for possible dynamic policyholder behaviour[50].

In this chapter, two ways of modeling dynamic lapses are introduced. Since the product has only one guarantee, dynamic lapses could be triggered by the value of the guaranteed death benefits. The model is designed according to the SOA approach. This way of modeling dynamic lapses is introduced for reasons of completeness since it is rather unlikely that policyholders would tie their lapse behavior to the guaranteed death benefits of a simple German unit-linked insurance. Dynamic lapses triggered by guarantees seem to be more important for insurance products with stronger guarantees such as accumulation or withdrawal guarantees. On the other hand, the product is very market sensitive, therefore it is also reasonable to model dynamic lapses triggered by the fund value. Very simple lapse functions are used in both cases, more sophisticated lapse functions can be found in the literature[51].

The impact of dynamic policyholder behavior on the solvency capital requirement is measured with the following approach: the output of a lapse function, denoted as the dynamic lapse multiplier, adjusts the deterministic lapse rates. The lapse rates therefore reflect a combination of irrational lapse bahaviour and rational lapse behavior. This setup makes sure that, when the lapse stress scenario is performed, only the deterministic lapse rates are affected directly while the risk from dynamic lapses is taken in account in the

[49]See Helfenstein & Barnshaw (2003, page 20), Hochreiter *et al.* (2007, page 8), Edwards (2009), Cerchiara *et al.* (2008) and Milliman (2009).

[50]See TS.II.D.11-15 CEIOPS (2008c, page 34).

[51]See Kochanski (2009), Kolkiewicz & Tan (2006), Smink (2001), Zenios (1999) and De Giovanni (2008).

sub-module of the trigger (here: the market risk at most)[52]. In the second step, the solvency capital requirement is recalculated with the average annual lapse rates of the first step just as if the insurer would experience lapses without the assumption of dynamic policyholder behavior. The impact of dynamic lapse rates is then the ratio of the SCR's obtained.

7.1 Dynamic lapse multiplier triggered by death benefits

The Society of Actuaries (SOA) has introduced a dynamic lapse multiplier for similar products (variable annuities – GMDB type). The multiplier adjusts the lapse rates depending on the ratio of the guaranteed death benefit and the current investment fund value. "This factor adjusts the lapse rate to reflect the antiselection present when the guarantee is in-the-money. Lapse rates may be lower when the guarantees have more value."[53] The lapse rates are adjusted with the following method: Let dlm_t denote the dynamic lapse multiplier, then

$$lr_t = lr_t^{det} \cdot dlm_t \qquad \text{with} \tag{84}$$

$$dlm_t = \min\left(1, \max\left(0.5, 1 - 1.25\left(\frac{DB_t}{FV_t} - 1.1\right)\right)\right) \tag{85}$$

7.2 Generalized dynamic lapse multiplier triggered by death benefits

In this setup, the SOA-multiplier is generalized to allow a variety of impact scenarios on the deterministic lapse rates:

$$lr_t = lr_t^{det} \cdot dlm_t \qquad \text{with} \tag{86}$$

$$dlm_t = \min\left(dlm_{max}, \max\left(dlm_{min}, 1 - adj_a\left(\frac{DB_t}{FV_t} - adj_b\right)\right)\right) \tag{87}$$

where dlm_{max} and dlm_{min} denote the maximum and the minimum value of the dynamic lapse multiplier, while adj_a and adj_b adjust the sensitivity of the multiplier to a change of the value of the guaranteed death benefits.

[52] This approach is presented in CEIOPS (2009a, page 20-24).
[53] See American Academy of Actuaries (2005, page 59).

7.3 Dynamic lapse multiplier triggered by the funds value

Another way to motivate dynamic policyholder behavior is to trigger dynamic lapse rates by the performance of the fund value. Using a simple step function and assuming that bad fund performance leads to higher lapse rates while good fund performance reduces lapses, the dynamic lapse multiplier can be defined as follows:

$$lr_t = lr_t^{det} \cdot dlm_t \qquad \text{with} \tag{88}$$

$$dlm_t = \begin{cases} dlm_{min}, & \text{for } \frac{A_t}{A_{\max\{0,t-d\}}} > adj_a \\ dlm_{max}, & \text{for } \frac{A_t}{A_{\max\{0,t-d\}}} < adj_b \\ 1, & \text{else} \end{cases} \tag{89}$$

where dlm_{max} and dlm_{min} denote the maximum and the minimum value of the dynamic lapse multiplier, while adj_a and adj_b set the fund value performance that triggers dynamic lapse behavior and d denotes the number of months the policyholder monitors the fund value until he makes a decision.

7.4 Parameter assumptions

Parameter	Value	Description	Category
dlm_{max}	1	maximum value of the dynamic lapse multiplier	Dynamic lapse multiplier triggered by death benefits
dlm_{min}	0.5	minimum value of the dynamic lapse multiplier	
adj_a	1.25	sensitivity factor a	
adj_b	1.1	sensitivity factor b	
dlm_{max}	1.5	maximum value of the dynamic lapse multiplier	Dynamic lapse multiplier triggered by the fund value
dlm_{min}	0.5	minimum value of the dynamic lapse multiplier	
adj_a	1.5	fund value performance triggering lower lapses	
adj_b	0.9	fund value performance triggering higher lapses	
d	12	monitoring period in months	

Table 7: Dynamic lapse model parameter assumptions

7.5 Numerical results

Simulations have been run for both kinds of lapse multipliers in order to detect the riskier dynamic lapse trigger.

Policy Type	P type	SCR	Profits	Solvency ratio	SCRmkt	SCRmort	SCRlapse	SCRexp
A	sin	-8,47%	-1,76%	7,33%	-9,31%	-11,52%	1,68%	-7,02%
B	sin	-0,94%	-0,37%	0,57%	-1,38%	-2,62%	0,05%	-0,94%
C	sin	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
D	sin	-2,20%	-0,51%	1,73%	-3,18%	-1,70%	0,17%	-1,93%
A	reg	-21,96%	-21,90%	0,08%	-23,86%	-27,04%	-20,92%	-26,54%
B	reg	-18,89%	-20,82%	-2,38%	-22,54%	-23,35%	-17,19%	-24,23%
C	reg	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
D	reg	-18,34%	-20,46%	-2,59%	-22,20%	-23,10%	-16,56%	-23,79%

Table 8: Numerical results - dynamic lapses triggered by death benefits

Policy Type	P type	SCR	Profits	Solvency ratio	SCRmkt	SCRmort	SCRlapse	SCRexp
A	sin	3,48%	1,18%	-2,22%	4,99%	10,52%	0,02%	7,01%
B	sin	3,59%	1,06%	-2,44%	5,48%	8,78%	0,04%	6,99%
C	sin	3,72%	1,04%	-2,59%	5,74%	2,83%	0,05%	6,97%
D	sin	3,68%	1,05%	-2,54%	5,67%	4,82%	0,05%	6,98%
A	reg	1,16%	5,98%	4,76%	5,92%	7,14%	-0,15%	6,82%
B	reg	1,13%	5,96%	4,78%	5,83%	6,69%	-0,13%	6,82%
C	reg	1,13%	5,97%	4,78%	5,81%	7,01%	-0,12%	6,82%
D	reg	1,13%	5,97%	4,78%	5,83%	7,09%	-0,12%	6,82%

Table 9: Numerical results - dynamic lapses triggered by the fund value

Tables 8 and 9 show substancial changes in the structure of the solvency capital requirement as well as in the profits due to dynamic lapses. Dynamic lapses triggered by death benefits lead to a decrease of the SCR_{mkt} for policies with strong guarantees such as policy type A. This is the only significant effect for single premium policies (note that SCR_{mort} and SCR_{exp} have only little influence on the SCR). For regular premium policies, there is also a substantial decrease in the profits as well as in SCR_{lapse}. Overall, the usage of dynamic lapses triggered by death benefits defined as in the model improves the solvency ratio for single premium policies and worsens the solvency ratio for regular premium policies.

Table 9 shows the more relevant results from simulations with dynamic lapses triggered by the fund value. Again, dynamic lapses lead to changes of the SCR_{mkt}, here the SCR_{mkt} increases, and since the type of the guarantee is not a trigger for dynamic lapses in this model, the changes are similar for all types of policies. For single premium policies, the SCR_{mkt} has a bigger influence on the SCR than for regular premium policies. The opposite effect occurs regarding the profits. Overall, the use of dynamic lapses triggered by the fund value defined as in the model worsens the solvency ratio for single premium policies and improves the solvency ratio for regular premium policies.

The changes of the deterministic lapses (run 1 – original deterministic lapses and run 2 – average overall lapses from run 1) are presented in tables 18 to 20[54].

The impact of dynamic lapse behavior as modeled in this paper on the solvency capital requirement of a German unit-linked insurance with guaranteed death benefits is not alarming. However, this may not be the case with unit-linked products with strong guarantees and options.

[54]See Appendix C.

8 Single equivalent scenario

The single equivalent scenario was developed to avoid double-counting of the loss-absorbing capacity of future discretionary benefits and to detect non-linearities[55]. As opposed to perform single stress tests to determine the solvency capital requirement for every risk module and then using the SCR-formulas, only one stress scenario is performed but with all stresses at one time and therefore with lesser shocks. The calibration of the shocks should be performed on the $BSCR$, therefore the derivation starts with the standard formula for basic solvency capital requirement:

$$BSCR = \sqrt{\sum_{i,j} \rho_{i,j} SCR_i SCR_j} \tag{90}$$

Since the stresses in the single equivalent scenario happen simultaneously, all correlation factors are changed to $\rho_{i,j} = 1,\ \forall i, j$[56]. The change of the correlation factors increases the level of significance. To ensure a constant level of significance, a change of correlation factors must be accompanied by an adjustment to the stress scenarios. Furthermore, linearity is assumed throughout the entire model, therefore, the solvency capital requirement for every sub-module can be adjusted by multiplying with a diversification reduction factor, which also is applied to the stress scenarios. Using the adjusted SCR_i and replacing the correlation factors, the $BSCR$ can be expressed as

$$\begin{aligned} BSCR &= \sqrt{\left(\sum_i drf_i\ SCR_i\right)^2} && (91)\\ &= \sqrt{\left(SCR_{int}^{SES} + SCR_{eq}^{SES} + SCR_{mort}^{SES} + SCR_{lapse}^{SES} + SCR_{exp}^{SES}\right)^2} && (92)\\ &= SCR_{int}^{SES} + SCR_{eq}^{SES} + SCR_{mort}^{SES} + SCR_{lapse}^{SES} + SCR_{exp}^{SES} && (93)\\ &= BSCR^{SES} && (94)\\ &= \Pi - \Pi|_{SES} \qquad \text{(without profit sharing)} && (95) \end{aligned}$$

where SCR_i^{SES} denotes the solvency capital requirement of the sub-module i resulting from an adjusted shock[57]. In QIS4, most shocks are expressed

[55] See CEIOPS (2009b) for general description and CEIOPS (2008b) for implementation in the standard formula.

[56] The single equivalent scenario method requires positive definite initial correlation matrices (see CEIOPS (2009a) for further information).

[57] $SCR_i^{SES} = drf_i\ SCR_i$ holds only with a linear model.

with factors to the relevant rates (e.g. mortality rates, lapse rates etc.), moderated shocks as used in the single equivalent scenario are created with diversification reduction factors. The diversification reduction factors that adjust the shock rates are derived through the following approach: The diversified solvency capital requirement is allocated to every sub-module with the covariance principle[58]. Then, the diversification reduction factors that adjust the stress scenarios are defined as the proportion of the allocated diversified solvency capital requirement to the stand-alone solvency capital requirement. Let C_{SCR}, C_{mkt} and C_{life} denote the correlation matrices of the overall SCR, the market risk and the underwriting risk modules. Let

$$U = \begin{pmatrix} SCR_{mkt} \\ SCR_{life} \end{pmatrix}, \qquad V = \begin{pmatrix} SCR_{int} \\ SCR_{eq} \end{pmatrix}, \qquad W = \begin{pmatrix} SCR_{mort} \\ SCR_{lapse} \\ SCR_{exp} \end{pmatrix}, \tag{96}$$

then the 1st step diversification reduction factors f are defined by:

$$\begin{pmatrix} f_{mkt} \\ f_{life} \end{pmatrix} = \frac{1}{BSCR} C_{SCR} U \tag{97}$$

$$\begin{pmatrix} f_{int} \\ f_{eq} \end{pmatrix} = \frac{1}{SCR_{mkt}} C_{mkt} V \tag{98}$$

$$\begin{pmatrix} f_{mort} \\ f_{lapse} \\ f_{exp} \end{pmatrix} = \frac{1}{SCR_{life}} C_{life} W \tag{99}$$

The 2nd step diversification reduction factors drf are obtained by multiplying the risk module 1st step diversification reduction factors with the overall 1st step diversification reduction factors, e.g. $drf_{mort} = f_{mort} f_{life}$ (the reduced mortality shock would be $10\% \cdot drf_{mort}$)[59].

Now, the reduced shocks can be used to calculate the net solvency capital requirement via the single equivalent scenario. A significant difference between the $nSCR$ and the $nSCR^{SES}$ suggests a significant double counting of loss-absorbing capacity of future discretionary benefits.

The existence of non-linearities leads to significant difference between the $BSCR$ and the $BSCR^{SES}$. Therefore, the single equivalent scenario can also be used to detect non-linearities.

[58] See Albrecht & Koryciorz (2004) for more information.

[59] See D for an example for above calculations.

Table 10 shows the $BSCR$ and the SCR obtained by the standard formula and the percental deviation of the $BSCR$ and the SCR obtained by the single equivalent scenario method. There is no indication for double counting of loss-absorbing capacity of future discretionary benefits since the deviation is almost identical for the $BSCR$ and the SCR. The deviation does not change with different bonus participation rates[60]. The reason for the difference of the solvency capital requirements is non-linearity[61]. The diversification reduction factors are presented in table 11. Nevertheless, requiring less computational capacities than the standard formula, the single equivalent scenario can be useful, once the diversification reduction factors are obtained. Unfortunately, the adjustment of the diversification reduction factors requires the calculation of the solvency capital requirement with the standard formula method. Therefore, the single equivalent scenario can not be used to replace the standard formula.

Policy Type	P type	BSCR	SCR	SES-BSCR	SES-SCR
A	sin	€ 28,98 mln	€ 15,03 mln	-4,55%	-4,83%
B	sin	€ 28,91 mln	€ 15,01 mln	-4,66%	-4,78%
C	sin	€ 29,58 mln	€ 15,17 mln	-4,87%	-4,88%
D	sin	€ 29,17 mln	€ 15,07 mln	-4,75%	-4,82%
A	reg	€ 27,46 mln	€ 14,76 mln	-6,41%	-6,27%
B	reg	€ 26,78 mln	€ 14,70 mln	-6,27%	-6,20%
C	reg	€ 26,84 mln	€ 14,75 mln	-6,25%	-6,18%
D	reg	€ 26,89 mln	€ 14,75 mln	-6,27%	-6,20%

Table 10: Numerical results - SES

Policy Type A		
premium type	sin	reg
drf_int	0,011	0,611
drf_eq	0,935	0,008
drf_mort	0,042	0,048
drf_lapse	0,571	0,917
drf_exp	0,346	0,508

Table 11: Diversification reduction factors - SES

[60] Tested with higher and lower bonus participation rates and also without minimum participation rates.

[61] As shown in chapter 6, figure 15.

Profits distribution – SES	Policy type A	P type sin			
	total	from risk	from expenses		
			total	from lapse	from kickbacks
with profit sharing					
Profits	€ 78,82 mln	€ 2,42 mln	€ 76,40 mln	€ 14,69 mln	€ 62,77 mln
Profits after combined shock	€ 51,16 mln	€ 2,39 mln	€ 48,77 mln	€ 12,49 mln	€ 37,50 mln
without profit sharing					
Profits	€ 40,10 mln	€ 0,60 mln	€ 39,50 mln	€ 7,41 mln	€ 32,62 mln
Profits after combined shock	€ 25,80 mln	€ 0,58 mln	€ 25,22 mln	€ 6,31 mln	€ 19,52 mln

Profits distribution – SES	Policy type A	P type reg			
	total	from risk	from expenses		
			total	from lapse	from kickbacks
with profit sharing					
Profits	€ 74,81 mln	€ 4,68 mln	€ 70,12 mln	€ 0,72 mln	€ 15,31 mln
Profits after combined shock	€ 49,10 mln	€ 3,08 mln	€ 46,03 mln	€ 0,88 mln	€ 10,04 mln
without profit sharing					
Profits	€ 40,15 mln	€ 1,11 mln	€ 39,04 mln	€ 0,37 mln	€ 8,52 mln
Profits after combined shock	€ 26,31 mln	€ 0,74 mln	€ 25,58 mln	€ 0,45 mln	€ 5,57 mln

Table 12: Composition of the profits – SES

9 Summary

The analysis reveals that market risk and lapse risk are in fact the main risks associated with a German unit-linked insurance product with guaranteed death benefits. Mortality and expense risks are negligible. The type of the death benefits has no impact on the solvency capital requirement. On the other hand, the premium type influences the type of market risks. The insurance product is linear to the risk factors for the most part. Some non-linearity has been revealed attached to lapse risks. This matter of fact causes a lower solvency capital requirement calculated with the single equivalent method. There is no indication of double-counting of the loss-absorbing capacity of future discretionary benefits so far. The single equivalent scenario method also proves to be a tool to review main assumptions of the standard formula. Dynamic policyholder behaviour has not a large impact on the solvency capital requirement for this particular insurance product. Nevertheless, dynamic lapses have a potential to be a major risk and should be evaluated with other unit-linked products and other lapse functions.

Appendices

A Interest rate shock

Maturity t (years)	1	2	3	4	5	6	7
relative change $s^{up}(t)$	0.94	0.77	0.69	0.62	0.56	0.52	0.49
relative change $s^{down}(t)$	-0.51	-0.47	-0.44	-0.42	-0.40	-0.38	-0.37

Maturity t (years)	8	9	10	11	12	13	14
relative change $s^{up}(t)$	0.46	0.44	0.42	0.42	0.42	0.42	0.42
relative change $s^{down}(t)$	-0.35	-0.34	-0.34	-0.34	-0.34	-0.34	-0.34

Maturity t (years)	15	16	17	18	19	20	20+
relative change $s^{up}(t)$	0.42	0.41	0.40	0.39	0.38	0.37	0.37
relative change $s^{down}(t)$	-0.34	-0.33	-0.33	-0.32	-0.31	-0.31	-0.31

Table 13: Interest rate shock

B Mortality tables

DAV-Sterbetafel 2008 T Männer 2,25%

x	1000 * q_x	l_x	d_x	D_x	N_x	C_x	M_x	S_x	R_x	x
0	6,113	1.000.000	6.113	1.000.000,000	36.300.634,756	5.978,484	201.208,526	1.024.816.900,952	13.749.651,606	0
1	0,423	993.887	420	972.016,626	35.300.634,756	402,115	195.230,042	988.516.266,196	13.548.443,079	1
2	0,343	993.467	341	950.225,392	34.328.618,130	318,755	194.827,927	953.215.631,440	13.353.213,037	2
3	0,275	993.126	273	928.997,031	33.378.392,739	249,853	194.509,171	918.887.013,309	13.158.385,111	3
4	0,220	992.853	218	908.304,701	32.449.395,708	195,430	194.259,319	885.508.620,571	12.963.875,939	4
5	0,182	992.634	181	888.122,126	31.541.091,006	158,081	194.063,889	853.059.224,863	12.769.616,621	5
6	0,155	992.454	154	868.421,015	30.652.968,880	131,643	193.905,808	821.518.133,857	12.575.552,732	6
7	0,139	992.300	138	849.179,863	29.784.547,865	115,439	193.774,164	790.865.164,977	12.381.646,924	7
8	0,129	992.162	128	830.378,315	28.935.368,002	104,762	193.658,726	761.080.617,112	12.187.872,760	8
9	0,125	992.034	124	812.001,170	28.104.989,687	99,267	193.553,964	732.145.249,109	11.994.214,034	9
10	0,129	991.910	128	794.033,907	27.292.988,517	100,176	193.454,697	704.040.259,422	11.800.660,070	10
11	0,143	991.782	142	776.461,102	26.498.954,611	108,591	193.354,521	676.747.270,905	11.607.205,373	11
12	0,173	991.640	172	759.266,570	25.722.493,509	128,463	193.245,930	650.248.316,294	11.413.850,852	12
13	0,222	991.469	220	742.430,530	24.963.226,939	161,193	193.117,468	624.525.822,785	11.220.604,922	13
14	0,303	991.248	300	725.932,235	24.220.796,409	215,117	192.956,275	599.562.595,846	11.027.487,454	14
15	0,417	990.948	413	709.743,059	23.494.864,175	289,450	192.741,157	575.341.799,436	10.834.531,180	15
16	0,557	990.535	552	693.835,790	22.785.121,116	377,962	192.451,707	551.846.935,262	10.641.790,022	16
17	0,709	989.983	702	678.190,048	22.091.285,325	470,256	192.073,745	529.061.814,146	10.449.338,315	17
18	0,850	989.281	841	662.796,295	21.413.095,278	550,980	191.603,489	506.970.528,821	10.257.264,570	18
19	0,953	988.440	942	647.660,555	20.750.298,983	603,639	191.052,509	485.557.433,543	10.065.661,081	19
20	1,012	987.498	999	632.805,217	20.102.638,428	626,307	190.448,870	464.807.134,560	9.874.608,572	20
21	1,022	986.499	1.008	618.254,101	19.469.833,211	617,952	189.822,563	444.704.496,132	9.684.159,702	21
22	1,004	985.491	989	604.031,536	18.851.579,110	593,103	189.204,612	425.234.662,921	9.494.337,138	22
23	0,963	984.501	948	590.146,786	18.247.547,574	555,806	188.611,509	406.383.083,812	9.305.132,526	23
24	0,911	983.553	896	576.604,865	17.657.400,788	513,728	188.055,703	388.135.536,238	9.116.521,018	24
25	0,856	982.657	841	563.403,010	17.080.795,923	471,661	187.541,975	370.478.135,450	8.928.465,314	25
26	0,808	981.816	793	550.533,728	16.517.392,913	435,043	187.070,314	353.397.339,527	8.740.923,340	26
27	0,772	981.023	757	537.984,251	15.966.859,185	406,185	186.635,272	336.879.946,613	8.553.853,025	27
28	0,752	980.265	737	525.739,782	15.428.874,934	386,657	186.229,087	320.913.087,428	8.367.217,754	28
29	0,745	979.528	730	513.784,280	14.903.135,152	374,346	185.842,430	305.484.212,494	8.180.988,667	29
30	0,752	978.799	736	502.104,167	14.389.350,872	369,274	185.468,084	290.581.077,342	7.995.146,236	30
31	0,768	978.063	751	490.686,146	13.887.246,706	368,554	185.098,810	276.191.726,469	7.809.678,153	31
32	0,791	977.311	773	479.520,097	13.396.560,560	370,954	184.730,256	262.304.479,764	7.624.579,342	32
33	0,820	976.538	801	468.597,356	12.917.040,463	375,794	184.359,302	248.907.919,204	7.439.849,087	33
34	0,855	975.738	834	457.910,128	12.448.443,107	382,898	183.983,507	235.990.878,741	7.255.489,785	34
35	0,895	974.903	873	447.450,968	11.990.532,979	391,656	183.600,609	223.542.435,634	7.071.506,278	35
36	0,945	974.031	920	437.213,203	11.543.082,010	404,075	183.208,953	211.551.902,656	6.887.905,668	36
37	1,005	973.110	978	427.188,299	11.105.868,808	419,877	182.804,878	200.008.820,646	6.704.696,715	37
38	1,083	972.132	1.053	417.368,191	10.678.680,508	442,063	182.385,001	188.902.951,838	6.521.891,837	38
39	1,181	971.079	1.147	407.741,986	10.261.312,317	470,947	181.942,938	178.224.271,329	6.339.506,836	39
40	1,301	969.933	1.262	398.298,722	9.853.570,331	506,784	181.471,991	167.962.959,012	6.157.563,898	40
41	1,447	968.671	1.402	389.027,418	9.455.271,609	550,536	180.965,207	158.109.388,681	5.976.091,907	41
42	1,623	967.269	1.570	379.916,377	9.066.244,191	603,036	180.414,671	148.654.117,072	5.795.126,700	42
43	1,833	965.699	1.770	370.953,323	8.686.327,814	664,995	179.811,635	139.587.872,881	5.614.712,029	43
44	2,082	963.929	2.007	362.125,541	8.315.374,491	737,355	179.146,640	130.901.545,067	5.434.900,394	44
45	2,364	961.922	2.274	353.419,653	7.953.248,950	817,099	178.409,285	122.586.170,577	5.255.753,753	45
46	2,669	959.648	2.561	344.825,594	7.599.829,296	900,088	177.592,186	114.632.921,627	5.077.344,468	46
47	2,983	957.087	2.855	336.337,657	7.255.003,703	981,218	176.692,098	107.033.092,331	4.899.752,282	47
48	3,302	954.232	3.151	327.955,366	6.918.666,046	1.059,079	175.710,881	99.778.088,628	4.723.060,184	48
49	3,630	951.081	3.452	319.679,665	6.590.710,680	1.134,902	174.651,801	92.859.422,583	4.547.349,303	49
50	3,981	947.629	3.773	311.510,247	6.271.031,015	1.212,834	173.516,899	86.268.711,902	4.372.697,502	50
51	4,371	943.856	4.126	303.442,665	5.959.520,768	1.297,162	172.304,066	79.997.680,887	4.199.180,602	51
52	4,812	939.731	4.522	295.468,281	5.656.078,104	1.390,507	171.006,904	74.038.160,119	4.026.876,536	52
53	5,308	935.209	4.964	287.576,027	5.360.609,823	1.492,864	169.616,397	68.382.082,015	3.855.869,632	53
54	5,857	930.244	5.448	279.755,084	5.073.033,797	1.602,470	168.123,533	63.021.472,192	3.686.253,235	54
55	6,460	924.796	5.974	271.996,634	4.793.278,713	1.718,434	166.521,063	57.948.438,395	3.518.129,702	55
56	7,117	918.822	6.539	264.292,944	4.521.282,079	1.839,582	164.802,630	53.155.159,682	3.351.608,639	56
57	7,831	912.283	7.144	256.637,625	4.256.989,135	1.965,505	162.963,047	48.633.877,603	3.186.806,009	57
58	8,604	905.138	7.788	249.024,837	4.000.351,510	2.095,462	160.997,542	44.376.888,468	3.023.842,962	58
59	9,454	897.351	8.484	241.449,611	3.751.326,673	2.232,435	158.902,080	40.376.536,958	2.862.845,420	59
60	10,404	888.867	9.248	233.904,104	3.509.877,062	2.379,989	156.669,645	36.625.210,285	2.703.943,340	60

Table 14: Mortality table I

DAV-Sterbetafel 2008 T Männer 2,25%

x	1000 * q_x	l_x	d_x	D_x	N_x	C_x	M_x	S_x	R_x	x
61	11,504	879.619	10.119	226.377,081	3.275.972,958	2.546,936	154.289,657	33.115.333,222	2.547.273,694	61
62	12,818	869.500	11.145	218.848,743	3.049.595,877	2.743,475	151.742,721	29.839.360,264	2.392.984,038	62
63	14,429	858.355	12.385	211.289,525	2.830.747,134	2.981,610	148.999,246	26.789.764,387	2.241.241,317	63
64	16,415	845.970	13.887	203.658,512	2.619.457,609	3.269,491	146.017,636	23.959.017,253	2.092.242,071	64
65	18,832	832.083	15.670	195.907,538	2.415.799,097	3.608,147	142.748,145	21.339.559,643	1.946.224,435	65
66	21,704	816.413	17.719	187.988,467	2.219.891,559	3.990,319	139.139,997	18.923.760,546	1.803.476,291	66
67	25,016	798.694	19.980	179.861,482	2.031.903,093	4.400,406	135.149,678	16.703.868,987	1.664.336,293	67
68	28,738	778.714	22.379	171.503,244	1.852.041,611	4.820,206	130.749,272	14.671.965,894	1.529.186,616	68
69	32,822	756.335	24.824	162.909,128	1.680.538,367	5.229,343	125.929,066	12.819.924,283	1.398.437,344	69
70	37,219	731.511	27.226	154.094,988	1.517.629,239	5.609,058	120.699,723	11.139.385,916	1.272.508,277	70
71	41,880	704.285	29.495	145.095,087	1.363.534,251	5.942,868	115.090,666	9.621.756,678	1.151.808,554	71
72	46,597	674.789	31.443	135.959,418	1.218.439,164	6.195,893	109.147,798	8.258.222,427	1.036.717,888	72
73	51,181	643.346	32.927	126.771,752	1.082.479,746	6.345,531	102.951,905	7.039.783,262	927.570,090	73
74	56,110	610.419	34.251	117.636,623	955.707,994	6.455,346	96.606,374	5.957.303,516	824.618,186	74
75	61,477	576.168	35.421	108.592,697	838.071,371	6.529,050	90.151,028	5.001.595,522	728.011,812	75
76	67,433	540.747	36.464	99.674,077	729.478,674	6.573,420	83.621,979	4.163.524,151	637.860,783	76
77	74,160	504.283	37.398	90.907,340	629.804,598	6.593,338	77.048,559	3.434.045,477	554.238,805	77
78	81,806	466.885	38.194	82.313,595	538.897,258	6.585,571	70.455,220	2.804.240,879	477.190,246	78
79	90,478	428.691	38.787	73.916,723	456.583,663	6.540,672	63.869,650	2.265.343,621	406.735,026	79
80	100,261	389.904	39.092	65.749,522	382.666,940	6.447,054	57.328,978	1.808.759,958	342.865,376	80
81	111,193	350.812	39.008	57.855,657	316.917,418	6.291,583	50.881,924	1.426.093,018	285.536,398	81
82	123,283	311.804	38.440	50.290,966	259.061,762	6.063,590	44.590,340	1.109.175,600	234.654,475	82
83	136,498	273.364	37.314	43.120,728	208.770,796	5.756,375	38.526,750	850.113,839	190.064,134	83
84	150,887	236.050	35.617	36.415,487	165.650,068	5.373,715	32.770,375	641.343,043	151.537,384	84
85	166,500	200.433	33.372	30.240,453	129.234,581	4.924,240	27.396,660	475.692,975	118.767,009	85
86	183,344	167.061	30.630	24.650,775	98.994,128	4.420,119	22.472,420	346.458,394	91.370,349	86
87	201,323	136.432	27.467	19.688,218	74.343,353	3.876,471	18.052,301	247.464,266	68.897,929	87
88	220,284	108.965	24.003	15.378,511	54.655,134	3.313,095	14.175,831	173.120,914	50.845,628	88
89	240,073	84.962	20.397	11.727,013	39.276,624	2.753,388	10.862,735	118.465,779	36.669,797	89
90	260,556	64.565	16.823	8.715,573	27.549,611	2.220,924	8.109,347	79.189,155	25.807,062	90
91	281,602	47.742	13.444	6.302,864	18.834,037	1.735,843	5.888,423	51.639,545	17.697,715	91
92	303,079	34.298	10.395	4.428,328	12.531,173	1.312,600	4.152,580	32.805,508	11.809,291	92
93	324,872	23.903	7.765	3.018,283	8.102,845	958,979	2.839,981	20.274,335	7.656,711	93
94	346,887	16.137	5.598	1.992,887	5.084,562	676,095	1.881,002	12.171,489	4.816,730	94
95	369,051	10.540	3.890	1.272,940	3.091,675	459,442	1.204,908	7.086,927	2.935,728	95
96	391,305	6.650	2.602	785,487	1.818,735	300,601	745,465	3.995,252	1.730,820	96
97	413,938	4.048	1.676	467,601	1.033,249	189,298	444,864	2.176,517	985,355	97
98	437,313	2.372	1.037	268,013	565,648	114,626	255,566	1.143,268	540,491	98
99	461,101	1.335	615	147,489	297,635	66,511	140,939	577,620	284,925	99
100	485,304	719	349	77,733	150,147	36,894	74,429	279,984	143,986	100
101	509,924	370	189	39,128	72,414	19,513	37,535	129,838	69,557	101
102	534,957	181	97	18,754	33,286	9,812	18,021	57,424	32,022	102
103	560,407	84	47	8,529	14,532	4,675	8,210	24,138	14,001	103
104	586,265	37	22	3,667	6,003	2,103	3,535	9,606	5,791	104
105	612,529	15	9	1,484	2,336	0,889	1,432	3,603	2,256	105
106	639,188	6	4	0,562	0,852	0,351	0,544	1,268	0,824	106
107	666,233	2	1	0,198	0,290	0,129	0,192	0,416	0,280	107
108	693,651	1	0	0,065	0,091	0,044	0,063	0,126	0,088	108
109	721,425	0	0	0,019	0,026	0,014	0,019	0,035	0,026	109
110	749,533	0	0	0,005	0,007	0,004	0,005	0,009	0,007	110
111	777,950	0	0	0,001	0,002	0,001	0,001	0,002	0,002	111
112	806,647	0	0	0,000	0,000	0,000	0,000	0,000	0,000	112
113	835,585	0	0	0,000	0,000	0,000	0,000	0,000	0,000	113
114	864,722	0	0	0,000	0,000	0,000	0,000	0,000	0,000	114
115	894,008	0	0	0,000	0,000	0,000	0,000	0,000	0,000	115
116	923,382	0	0	0,000	0,000	0,000	0,000	0,000	0,000	116
117	952,778	0	0	0,000	0,000	0,000	0,000	0,000	0,000	117
118	982,113	0	0	0,000	0,000	0,000	0,000	0,000	0,000	118
119	1000,000	0	0	0,000	0,000	0,000	0,000	0,000	0,000	119
120	1000,000	0	0	0,000	0,000	0,000	0,000	0,000	0,000	120
121	1000,000	0	0	0,000	0,000	0,000	0,000	0,000	0,000	121

Table 15: Mortality table II

DAV-Sterbetafel 2008 T Frauen 2,25%

y	1000 * q_y	l_y	d_y	D_y	N_y	C_y	M_y	S_y	R_y	y
0	5,088	1.000.000	5.088	1.000.000,000	37.323.540,567	4.976,039	178.699,596	1.095.402.631,223	13.219.326,188	0
1	0,387	994.912	385	973.019,071	36.323.540,567	368,272	173.723,557	1.058.079.090,656	13.040.626,592	1
2	0,318	994.527	316	951.239,621	35.350.521,496	295,838	173.355,285	1.021.755.550,089	12.866.903,035	2
3	0,255	994.211	254	930.011,860	34.399.281,875	231,934	173.059,447	986.405.028,592	12.693.547,750	3
4	0,202	993.957	201	909.315,117	33.469.270,015	179,640	172.827,513	952.005.746,717	12.520.488,303	4
5	0,163	993.756	162	889.126,098	32.559.954,898	141,738	172.647,873	918.536.476,702	12.347.660,790	5
6	0,134	993.594	133	869.419,238	31.670.828,800	113,939	172.506,134	885.976.521,803	12.175.012,917	6
7	0,115	993.461	114	850.173,824	30.801.409,563	95,619	172.392,196	854.305.693,003	12.002.506,783	7
8	0,105	993.347	104	831.370,224	29.951.235,739	85,373	172.296,577	823.504.283,440	11.830.114,587	8
9	0,099	993.243	98	812.990,641	29.119.865,514	78,715	172.211,204	793.553.047,701	11.657.818,010	9
10	0,102	993.144	101	795.022,156	28.306.874,873	79,308	172.132,489	764.433.182,187	11.485.606,805	10
11	0,111	993.043	110	777.448,474	27.511.852,717	84,398	172.053,181	736.126.307,314	11.313.474,316	11
12	0,127	992.933	126	760.256,408	26.734.404,243	94,428	171.968,784	708.614.454,597	11.141.421,135	12
13	0,153	992.807	152	743.432,621	25.974.147,836	111,242	171.874,356	681.880.050,354	10.969.452,351	13
14	0,188	992.655	187	726.962,226	25.230.715,215	133,662	171.763,113	655.905.902,518	10.797.577,995	14
15	0,228	992.468	226	710.831,840	24.503.752,989	158,503	171.629,452	630.675.187,304	10.625.814,882	15
16	0,271	992.242	269	695.031,561	23.792.921,148	184,209	171.470,949	606.171.434,315	10.454.185,430	16
17	0,310	991.973	308	679.553,259	23.097.889,588	206,026	171.286,740	582.378.513,167	10.282.714,481	17
18	0,324	991.666	321	664.393,738	22.418.336,329	210,527	171.080,714	559.280.623,579	10.111.427,742	18
19	0,330	991.344	327	649.563,300	21.753.942,591	209,639	170.870,187	536.862.287,250	9.940.347,028	19
20	0,328	991.017	325	635.060,092	21.104.379,290	203,716	170.660,548	515.108.344,659	9.769.476,841	20
21	0,322	990.692	319	620.881,949	20.469.319,198	195,525	170.456,832	494.003.965,369	9.598.816,293	21
22	0,314	990.373	311	607.023,985	19.848.437,249	186,411	170.261,307	473.534.646,171	9.428.359,461	22
23	0,304	990.062	301	593.480,078	19.241.413,264	176,448	170.074,896	453.686.208,922	9.258.098,153	23
24	0,297	989.761	294	580.244,166	18.647.933,186	168,540	169.898,448	434.444.795,658	9.088.023,257	24
25	0,293	989.467	290	567.307,417	18.067.689,020	162,563	169.729,908	415.796.862,472	8.918.124,809	25
26	0,292	989.177	289	554.661,316	17.500.381,603	158,397	169.567,344	397.729.173,452	8.748.394,901	26
27	0,292	988.888	289	542.297,658	16.945.720,287	154,866	169.408,947	380.228.791,849	8.578.827,557	27
28	0,296	988.600	293	530.209,591	16.403.422,629	153,489	169.254,081	363.283.071,562	8.409.418,610	28
29	0,302	988.307	298	518.388,899	15.873.213,038	153,109	169.100,592	346.879.648,933	8.240.164,529	29
30	0,311	988.009	307	506.828,700	15.354.824,139	154,155	168.947,484	331.006.435,894	8.071.063,936	30
31	0,327	987.701	323	495.521,835	14.847.995,440	158,470	168.793,329	315.651.611,755	7.902.116,453	31
32	0,351	987.378	347	484.459,461	14.352.473,605	166,303	168.634,858	300.803.616,315	7.733.323,124	32
33	0,386	987.032	381	473.632,680	13.868.014,144	178,799	168.468,555	286.451.142,710	7.564.688,266	33
34	0,433	986.651	427	463.031,646	13.394.381,464	196,081	168.289,756	272.583.128,565	7.396.219,711	34
35	0,490	986.224	483	452.646,605	12.931.349,818	216,916	168.093,675	259.188.747,101	7.227.929,955	35
36	0,555	985.740	547	442.469,250	12.478.703,213	240,167	167.876,759	246.257.397,284	7.059.836,280	36
37	0,624	985.193	615	432.492,596	12.036.233,963	263,937	167.636,592	233.778.694,071	6.891.959,521	37
38	0,701	984.578	690	422.711,707	11.603.741,367	289,800	167.372,655	221.742.460,108	6.724.322,929	38
39	0,783	983.888	770	413.120,182	11.181.029,659	316,355	167.082,855	210.138.718,742	6.556.950,274	39
40	0,872	983.118	857	403.713,163	10.767.909,477	344,291	166.766,500	198.957.689,082	6.389.867,419	40
41	0,972	982.261	955	394.485,208	10.364.196,314	375,002	166.422,208	188.189.779,605	6.223.100,919	41
42	1,084	981.306	1.064	385.429,602	9.969.711,106	408,612	166.047,206	177.825.583,291	6.056.678,711	42
43	1,213	980.242	1.189	376.539,654	9.584.281,504	446,692	165.638,594	167.855.872,185	5.890.631,505	43
44	1,359	979.053	1.331	367.807,249	9.207.741,849	488,851	165.191,902	158.271.590,681	5.724.992,910	44
45	1,524	977.723	1.490	359.224,840	8.839.934,601	535,412	164.703,051	149.063.848,831	5.559.801,008	45
46	1,706	976.232	1.665	350.784,725	8.480.709,761	585,270	164.167,640	140.223.914,231	5.395.097,956	46
47	1,903	974.567	1.855	342.480,475	8.129.925,036	637,399	163.582,369	131.743.204,470	5.230.930,317	47
48	2,109	972.712	2.051	334.306,831	7.787.444,561	689,538	162.944,971	123.613.279,433	5.067.347,948	48
49	2,324	970.661	2.256	326.260,908	7.453.137,730	741,546	162.255,432	115.825.834,872	4.904.402,977	49
50	2,546	968.405	2.466	318.340,027	7.126.876,822	792,659	161.513,886	108.372.697,143	4.742.147,545	50
51	2,782	965.940	2.687	310.542,331	6.808.536,795	844,918	160.721,228	101.245.820,321	4.580.633,658	51
52	3,035	963.252	2.923	302.863,963	6.497.994,464	898,965	159.876,309	94.437.283,526	4.419.912,431	52
53	3,306	960.329	3.175	295.300,509	6.195.130,502	954,781	158.977,344	87.939.289,061	4.260.036,121	53
54	3,593	957.154	3.439	287.847,673	5.899.829,993	1.011,478	158.022,563	81.744.158,559	4.101.058,777	54
55	3,898	953.715	3.718	280.502,138	5.611.982,320	1.069,337	157.011,085	75.844.328,567	3.943.036,214	55
56	4,228	949.997	4.017	273.260,382	5.331.480,181	1.129,922	155.941,747	70.232.346,247	3.786.025,129	56
57	4,585	945.981	4.337	266.117,396	5.058.219,799	1.193,299	154.811,826	64.900.866,066	3.630.083,382	57
58	4,974	941.643	4.684	259.068,213	4.792.102,403	1.260,250	153.618,527	59.842.646,267	3.475.271,556	58
59	5,402	936.960	5.061	252.107,196	4.533.034,190	1.331,915	152.358,277	55.050.543,864	3.321.653,029	59
60	5,884	931.898	5.483	245.227,690	4.280.926,994	1.411,168	151.026,362	50.517.509,673	3.169.294,752	60

Table 16: Mortality table III

DAV-Sterbetafel 2008 T Frauen 2,25%

y	1000 * q_y	l_y	d_y	D_y	N_y	C_y	M_y	S_y	R_y	y
61	6,449	926.415	5.974	238.420,313	4.035.699,305	1.503,738	149.615,194	46.236.582,679	3.018.268,390	61
62	7,126	920.441	6.559	231.670,162	3.797.278,992	1.614,554	148.111,455	42.200.883,374	2.868.653,196	62
63	7,935	913.881	7.252	224.957,731	3.565.608,830	1.745,760	146.496,901	38.403.604,382	2.720.541,741	63
64	8,898	906.630	8.067	218.261,801	3.340.651,099	1.899,358	144.751,141	34.837.995,552	2.574.044,840	64
65	10,025	898.563	9.008	211.559,616	3.122.389,298	2.074,215	142.851,783	31.497.344,453	2.429.293,699	65
66	11,323	889.555	10.072	204.830,055	2.910.829,682	2.268,255	140.777,568	28.374.955,155	2.286.441,916	66
67	12,797	879.482	11.255	198.054,537	2.705.999,627	2.478,732	138.509,313	25.464.125,473	2.145.664,348	67
68	14,460	868.227	12.555	191.217,636	2.507.945,090	2.704,163	136.030,580	22.758.125,845	2.007.155,035	68
69	16,332	855.673	13.975	184.305,750	2.316.727,454	2.943,845	133.326,417	20.250.180,755	1.871.124,455	69
70	18,440	841.698	15.521	177.306,277	2.132.421,704	3.197,582	130.382,572	17.933.453,301	1.737.798,037	70
71	20,813	826.177	17.195	170.207,090	1.955.115,427	3.464,567	127.184,990	15.801.031,597	1.607.415,465	71
72	23,475	808.982	18.991	162.997,134	1.784.908,337	3.742,159	123.720,423	13.845.916,170	1.480.230,475	72
73	27,035	789.991	21.357	155.668,241	1.621.911,203	4.115,884	119.978,263	12.061.007,832	1.356.510,053	73
74	30,413	768.634	23.376	148.126,895	1.466.242,962	4.405,852	115.862,380	10.439.096,629	1.236.531,789	74
75	34,287	745.257	25.553	140.461,527	1.318.116,067	4.710,029	111.456,528	8.972.853,667	1.120.669,409	75
76	38,749	719.704	27.888	132.660,658	1.177.654,540	5.027,352	106.746,500	7.654.737,600	1.009.212,881	76
77	43,937	691.817	30.396	124.714,123	1.044.993,882	5.358,987	101.719,147	6.477.083,060	902.466,382	77
78	49,993	661.420	33.066	116.610,815	920.279,759	5.701,442	96.360,160	5.432.089,178	800.747,234	78
79	57,024	628.354	35.831	108.343,365	803.668,944	6.042,222	90.658,718	4.511.809,419	704.387,074	79
80	65,113	592.523	38.581	99.917,059	695.325,580	6.362,738	84.616,496	3.708.140,475	613.728,356	80
81	74,288	553.942	41.151	91.355,657	595.408,521	6.637,290	78.253,758	3.012.814,895	529.111,861	81
82	84,590	512.791	43.377	82.708,096	504.052,864	6.842,326	71.616,468	2.417.406,374	450.858,102	82
83	96,095	469.414	45.108	74.045,788	421.344,768	6.958,856	64.774,143	1.913.353,511	379.241,634	83
84	109,028	424.305	46.261	65.457,563	347.298,980	6.979,665	57.815,287	1.492.008,743	314.467,492	84
85	123,611	378.044	46.730	57.037,512	281.841,417	6.895,319	50.835,622	1.144.709,763	256.652,205	85
86	140,022	331.314	46.391	48.887,088	224.803,906	6.694,638	43.940,303	862.868,345	205.816,583	86
87	158,257	284.922	45.091	41.116,695	175.916,818	6.363,819	37.245,664	638.064,439	161.876,280	87
88	178,185	239.832	42.734	33.848,108	134.800,123	5.898,509	30.881,846	462.147,621	124.630,616	88
89	199,669	197.097	39.354	27.204,775	100.952,016	5.312,421	24.983,337	327.347,498	93.748,770	89
90	222,504	157.743	35.098	21.293,716	73.747,240	4.633,679	19.670,916	226.395,483	68.765,433	90
91	246,453	122.645	30.226	16.191,471	52.453,524	3.902,628	15.037,237	152.648,242	49.094,517	91
92	271,195	92.418	25.063	11.932,552	36.262,053	3.164,840	11.134,610	100.194,718	34.057,279	92
93	295,584	67.355	19.909	8.505,138	24.329,501	2.458,663	7.969,770	63.932,665	22.922,670	93
94	319,362	47.446	15.152	5.859,321	15.824,363	1.830,068	5.511,107	39.603,164	14.952,900	94
95	343,441	32.294	11.091	3.900,319	9.965,043	1.310,053	3.681,039	23.778,800	9.441,793	95
96	367,818	21.203	7.799	2.504,440	6.064,724	900,908	2.370,986	13.813,758	5.760,753	96
97	392,493	13.404	5.261	1.548,422	3.560,284	594,372	1.470,079	7.749,034	3.389,767	97
98	417,460	8.143	3.399	919,978	2.011,862	375,603	875,707	4.188,750	1.919,689	98
99	442,716	4.744	2.100	524,131	1.091,884	226,935	500,104	2.176,889	1.043,982	99
100	468,258	2.644	1.238	285,662	567,753	130,820	273,169	1.085,005	543,878	100
101	494,075	1.406	695	148,556	282,091	71,783	142,349	517,252	270,709	101
102	520,164	711	370	73,504	133,534	37,393	70,566	235,161	128,360	102
103	546,514	341	186	34,494	60,030	18,437	33,173	101,627	57,794	103
104	573,114	155	89	15,298	25,536	8,575	14,736	41,597	24,621	104
105	599,953	66	40	6,387	10,238	3,748	6,162	16,061	9,884	105
106	627,014	26	17	2,499	3,851	1,532	2,414	5,823	3,723	106
107	654,283	10	6	0,912	1,352	0,583	0,882	1,972	1,309	107
108	681,741	3	2	0,308	0,440	0,205	0,299	0,620	0,427	108
109	709,364	1	1	0,096	0,132	0,067	0,093	0,180	0,128	109
110	737,130	0	0	0,027	0,036	0,020	0,026	0,048	0,035	110
111	765,011	0	0	0,007	0,009	0,005	0,007	0,011	0,009	111
112	792,974	0	0	0,002	0,002	0,001	0,002	0,002	0,002	112
113	820,987	0	0	0,000	0,000	0,000	0,000	0,000	0,000	113
114	849,009	0	0	0,000	0,000	0,000	0,000	0,000	0,000	114
115	876,998	0	0	0,000	0,000	0,000	0,000	0,000	0,000	115
116	904,905	0	0	0,000	0,000	0,000	0,000	0,000	0,000	116
117	932,675	0	0	0,000	0,000	0,000	0,000	0,000	0,000	117
118	960,249	0	0	0,000	0,000	0,000	0,000	0,000	0,000	118
119	987,564	0	0	0,000	0,000	0,000	0,000	0,000	0,000	119
120	1000,000	0	0	0,000	0,000	0,000	0,000	0,000	0,000	120
121	1000,000	0	0	0,000	0,000	0,000	0,000	0,000	0,000	121

Table 17: Mortality table IV

C Deterministic lapse changes

policy year	det. lapse Run 1	det. lapse Run 2
1	10,00%	9,48%
2	9,00%	8,19%
3	8,00%	7,16%
4	7,00%	6,20%
5	6,00%	5,28%
6	5,00%	4,38%
7	4,00%	3,50%
8	3,00%	2,62%
9	2,00%	1,74%
10	2,00%	1,74%
11	2,00%	1,73%
12	2,00%	1,73%
13	2,00%	1,73%
14	2,00%	1,72%
15	2,00%	1,72%
16	2,00%	1,72%
17	2,00%	1,72%
18	2,00%	1,72%
19	2,00%	1,72%
20	2,00%	1,72%
21	2,00%	1,72%
22	2,00%	1,72%
23	2,00%	1,72%
24	2,00%	1,72%
25	2,00%	1,72%
26	2,00%	1,72%
27	2,00%	1,73%
28	2,00%	1,73%
29	2,00%	1,73%
30	2,00%	1,73%

Table 18: Change of deterministic lapses - dynamic lapses triggered by death benefits - single premium policy type A

policy year	det. lapse Run 1	det. lapse Run 2
1	10,00%	5,12%
2	9,00%	4,60%
3	8,00%	4,08%
4	7,00%	3,56%
5	6,00%	3,04%
6	5,00%	2,53%
7	4,00%	2,02%
8	3,00%	1,51%
9	2,00%	1,01%
10	2,00%	1,01%
11	2,00%	1,02%
12	2,00%	1,03%
13	2,00%	1,06%
14	2,00%	1,10%
15	2,00%	1,14%
16	2,00%	1,18%
17	2,00%	1,24%
18	2,00%	1,29%
19	2,00%	1,34%
20	2,00%	1,39%
21	2,00%	1,44%
22	2,00%	1,48%
23	2,00%	1,52%
24	2,00%	1,55%
25	2,00%	1,59%
26	2,00%	1,62%
27	2,00%	1,65%
28	2,00%	1,67%
29	2,00%	1,70%
30	2,00%	1,72%

Table 19: Change of deterministic lapses - dynamic lapses triggered by death benefits - regular premium policy type A

policy year	det. lapse Run 1	det. lapse Run 2
1	10,00%	10,72%
2	9,00%	9,94%
3	8,00%	8,85%
4	7,00%	7,75%
5	6,00%	6,65%
6	5,00%	5,53%
7	4,00%	4,42%
8	3,00%	3,31%
9	2,00%	2,22%
10	2,00%	2,22%
11	2,00%	2,21%
12	2,00%	2,21%
13	2,00%	2,21%
14	2,00%	2,22%
15	2,00%	2,22%
16	2,00%	2,21%
17	2,00%	2,21%
18	2,00%	2,21%
19	2,00%	2,22%
20	2,00%	2,21%
21	2,00%	2,21%
22	2,00%	2,22%
23	2,00%	2,21%
24	2,00%	2,22%
25	2,00%	2,21%
26	2,00%	2,21%
27	2,00%	2,21%
28	2,00%	2,22%
29	2,00%	2,21%
30	2,00%	2,21%

Table 20: Change of deterministic lapses - dynamic lapses triggered by the fund value - single premium policy type A

D Example SES

		corr matrix		1st step diversification factor	2nd step diversification factor
SCRint	10	1	0	$\frac{10 \cdot 1 + 20 \cdot 0}{22.36} = 0.45$	$0.45 \cdot 0.46 = 0.21$
SCReq	20	0	1	$\frac{10 \cdot 0 + 20 \cdot 1}{22.36} = 0.89$	$0.89 \cdot 0.46 = 0.41$
diversified capital = SCRmkt	22.36				

		corr matrix			1st step diversification factor	2nd step diversification factor
SCRmort	30	1	0	0.25	$\frac{30 \cdot 1 + 40 \cdot 0 + 50 \cdot 0.25}{88.03} = 0.48$	$0.48 \cdot 0.97 = 0.47$
SCRlapse	40	0	1	0.5	$\frac{30 \cdot 0 + 40 \cdot 1 + 50 \cdot 0.5}{88.03} = 0.74$	$0.74 \cdot 0.97 = 0.72$
SCRexp	50	0.25	0.5	1	$\frac{30 \cdot 0.25 + 40 \cdot 0.5 + 50 \cdot 1}{88.03} = 0.88$	$0.88 \cdot 0.97 = 0.86$
diversified capital = SCRlife	88.03					

		corr matrix		1st step diversification factor
SCRmkt	22.36	1	0.25	$\frac{22.36 \cdot 1 + 88.03 \cdot 0.25}{96.10} = 0.46$
SCRlife	88.03	0.25	1	$\frac{22.36 \cdot 0.25 + 88.03 \cdot 1}{96.10} = 0.97$
diversified capital = SCR	96.10			

Table 21: Single equivalent scenario – example

References

ALBRECHT, PETER, & KORYCIORZ, SVEN. 2004. Methoden der risikobasierten Kapitalallokation im Versicherungs- und Finanzwesen. *Zeitschrift für die gesamte Versicherungswissenschaft*, **93**, 123–160.

ALFONSI, AURÉLIEN. 2006. *Discretization schemes for the Cox-Ingersoll-Ross processes.*

AMERICAN ACADEMY OF ACTUARIES. 2005. *Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products.*

ANZILLI, LUCA, & DE CESARE, LUIGI. 2007. Valuation of the surrender option in unit-linked life insurance policies in a non-rational behaviour framework. *Quaderni DSEMS*, **20**.

BACINELLO, ANNA RITA. 2001. Fair pricing of life insurance participating policies with a minimum interest rate guarantee. *ASTIN Bulletin*, **31(2)**, 275–297.

BACINELLO, ANNA RITA. 2003. Fair valuation of a guaranteed life insurance participating contract embedding a surrender option. *Journal of Risk and Insurance*, **70**, 461–487.

BAUER, DANIEL, KIESEL, RÜDIGER, KLING, ALEXANDER, & RUSS, JOCHEN. 2006. Risk-neutral valuation of participating life insurance contracts. *Insurance: Mathematics and Economics*, **39(2)**, 171–183.

BAUER, DANIEL, KLING, ALEXANDER, & RUSS, JOCHEN. 2008. A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities. *ASTIN Bulletin*, **38(2)**, 621 – 651.

BAUER, DANIEL, BERGMANN, DANIELA, & REUSS, ANDREAS. 2009. *Solvency II and Nested Simulations: a Least-Squares Monte Carlo Approach.*

BUNDESMINISTERIUM DER JUSTIZ. 2009. *Verordnung über die Kapitalausstattung von Versicherungsunternehmen.*

CEIOPS. 2008a. *QIS4 Report.*

CEIOPS. 2008b. *QIS4 Solo Spreadsheet (revised 31 July).* http://www.ceiops.eu/media/docman/QIS4

CEIOPS. 2008c. *QIS4 Technical Specifications.*

CEIOPS. 2009a. *CEIOPS Advice for Level 2 Implementing Measures on Sollvency II: SCR standard formula Loss-absorbing capacity of technical provisions and deferred taxes (former CP 54).*

CEIOPS. 2009b. *CEIOPS Advice for Level 2 Implementing Measures on Sollvency II: Standard formula SCR - Article 109 c Life underwriting risk (former CP 49).*

Cerchiara, Rocco Roberto, Edwards, Matthew, & Gambini, Alessandra. 2008. Generalized linear models in life insurance: decrements and risk factor analysis under Solvency II. *18th International AFIR Colloquium.*

Cox, Samuel H., & Lin, Yijia. 2006. *Annuity Lapse Rate Modeling: Tobit or not Tobit.* Society of Actuaries.

Daniel, James, & Vaaler, Leslie. 2007. *Mathematical interest theory.* Pearson Prentice Hall.

DAV-Arbeitsgruppe Stornoabzüge. 2007. *Stornoabzüge in der Lebensversicherung nach VVG-Neuregelung (Stand 20.12.2006).*

De Giovanni, Domenico. 2008. Lapse Rate Modeling: A Rational Expectation Approach. *Scandinavian Actuarial Journal.*

Doff, René. 2008. A Critical Analysis of the Solvency II Proposals. *The Geneva Papers*, **33**, 193–206.

Duverne, Denis, & Le Douit, Jacques. 2009. IFRS Phase II and Solvency II: Key Issues, Current Debates. *The Geneva Papers*, **34**, 47–55.

Edwards, Matthew. 2009. *Policyholder behaviour and advanced retention analysis.*

Elderfield, Matthew. 2009. Solvency II: Setting the Pace for Regulatory Change. *The Geneva Papers*, **34**, 35–41.

Enz, Rudolf. 2006. World insurance in 2005: moderate premium growth, attractive profitability. *SwissRe Sigma*, **5**.

Gatzert, Nadine. 2009. Implicit options in life insurance: An overview. *Zeitschrift für die gesamte Versicherungswissenschaft*, **98**, 141–164.

GDV. 2005. *Diskussionsbeitrag für einen Solvency II kompatiblen Standardansatz.*

GROSEN, ANDERS, & JORGENSEN, PETER. 2000. Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies. *Insurance: Mathematics and Economics*, **26**, 37–57.

HELFENSTEIN, RAINER, & BARNSHAW, MIKE. 2003. Unit-linked life insurance in Western Europe: regaining momentum? *SwissRe Sigma*, **3**.

HOCHREITER, RONALD, PFLUG, GEORG, & PAULSEN, VOLKERT. 2007. Design and management of unit-linked life insurance contracts with guarantees. *Handbook of Asset and Liability Management*, **2**, 627–662.

HOLZMÜLLER, INES. 2009. The United States RBC Standards, Solvency II and the Swiss Solvency Test: A Comparative Assessment. *The Geneva Papers*, **34**, 56–77.

KOCHANSKI, MICHAEL. 2009. *Berechnung des Solvenzkapitals für eine vereinfachte fondsgebundene Lebensversicherung.*

KOLKIEWICZ, ADAM W., & TAN, KEN SENG. 2006. Unit-linked life insurance contracts with lapse rates dependent on economic factors. *Annals of actuarial science*, **1**, 49–78.

KUO, WEIYU, TSAI, CHENGSIEN, & CHEN, WEI-KUANG. 2003. An empirical study on the lapse rate: the cointegration approach. *The Journal of Risk and Insurance*, **70**(3), 489–508.

MAUER, LAURENCE, & HOLDEN, NEIL. 2007. Determinants of the lapse rate in life insurance operating companies. *Review of Business*, **28**(1), 54–64.

MILLIMAN. 2009. *Variable Annuities, MCC Workshop VAs.*

MÜLLER, HELMUT. 1997. *Solvabilität der Versicherungsunternehmen.* Konferenz der Versicherungsaufsichtsbehörde der Mitgliedsstaaten der Europäischen Union.

MÄRTEN, GESINE, & DAALMANN, STEFANIE. 2009. *Fondsgebundene Versicherungsprodukte mit Garantien trotzen der Finanzmarktkrise.* Towers Perrin.

NORDAHL, HELGE. 2008. Valuation of life insurance surrender and exchange options. *Insurance: Mathematics and Economics*, **42**, 909–919.

Outreville, J. Francois. 1990. Whole-life insurance lapse rates and the emergency fund hypothesis. *Insurance: Mathematics and Economics*, **9**, 249–255.

Prestele, Andreas. 2006. *Storno in der Lebensversicherung: Einflussfaktoren, Auswirkungen, Gegenmassnahmen*. Vdm Verlag Dr. Müller.

Shreve, Steven E. 2000. *Stochastic Calculus for Finance 2*. Springer.

Smink, Meye. 2001. Risk Measurment for Asset Liability Matching, A simulation Approach to single premium deferred Annuities. *2nd AFIR Colloquium*, **2**, 75–92.

Steffen, Thomas. 2008. Solvency II and the Work of CEIOPS. *The Geneva Papers*, **33**, 60–65.

Steffensen, Mogens. 2002. Intervention options in life insurance. *Insurance: Mathematics and Economics*, **31(1)**, 71–85.

Zenios, Stavros A. 1999. *Financial Optimization*. Cambridge University Press.