Assessing Investment and Longevity Risks within Immediate Annuities

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Abstract

Life annuities provide a guaranteed income for the remainder of the recipient's lifetime, and therefore, annuitization presents an important option when choosing an adequate investment strategy for the retirement ages. While there are numerous research articles studying annuities from a pensioner's point of view, thus far there have been few contributions considering annuities from the provider's perspective. In particular, there are no surveys of the general risks within annuity books.

The present paper aims at filling this gap: Using a simulation framework, it provides a long-term analysis of the risks within annuity books. In particular, the joint impact of systematic mortality risk and investment risk as well as their respective influences on the insurer's financial situation are studied.

The key finding is that, under the model specifications and using annuity data from the United Kingdom, the risk premium charged for aggregate mortality or, more precisely, longevity risk seems to be very large relative to its characteristics. Possible explanations as well as economic implications are provided, and potential caveats are discussed.

Keywords: Annuities, Lee-Carter Model, Longevity Risk

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1 Introduction

Life annuities provide a guaranteed income for the remainder of the recipient's lifetime; therefore, annuitization presents an important option when choosing an adequate investment strategy for the retirement ages. Yaari (1965) showed that in a deterministic financial economy and in the absence of bequest motives, expected utility maximizers will annuitize their entire wealth. The same result was recently confirmed by Davidoff et al. (2005) in a more general market setting. However, although the current problems with state-run *pay-as-you-go* schemes in many countries and the shift from defined benefit to defined contribution plans should increase the demand for annuities, empirical studies show that the proportion of retirees choosing to annuitize their entire wealth or part thereof is rather small. In economic literature this phenomenon is commonly referred to as the "annuity puzzle".

Possible explanations for this peculiarity may be the consumption limitations or, more precisely, the suboptimal consumption profile due to constant annuity pay-outs (see, for example, Brown (2001) or Milevsky et al. (2006)), demographic risk (Schulze and Post (2006)), or the existence of bequest motives. Another reason could be that annuities are overpriced or, more specifically, are conceived to be overpriced by investors: Typically, yields on long-term debt are used to calculate the *actuarially fair price* of an annuity, even though the rate of return on, for example, the insurer's capital may be substantially higher (see James and Song (2001)). Therefore, even if annuities are priced *actuarially fair* – we have not yet specified what this means exactly – their prices may still be regarded as too high by investors.

In Mitchell et al. (1999), the *actuarially fair price* of an annuity is defined to be the expected present value, where current yields of treasury and corporate bonds as well as cohort mortality tables that are adjusted for selection effects[1] are used to determine interest and mortality rates, respectively. They find that in the US in 1995 the average annuity policy generally delivered pay-outs of less than 91 cents per unit of annuity premium. According to the authors, this "transaction cost" is primarily due to expenses, profit margins, and contingency funds. However, the question of whether this discount is adequate regarding the inherent risk is not addressed. Similar results are found in Finkelstein and Poterba (2002) for the United Kingdom (UK) with the same notion of *actuarially fair* priced annuities. Here, the difference between the actual and the fair value per unit of annuity premium, which amounts to between 5 and 14 cents for different types of annuity contracts and different cohorts, is not analyzed further either.

Murthi et al. (1999) come to similar conclusions. However, they note that annuitization costs of about 5% of the account value are very small in comparison to typical costs during the accumulation phase of a personal pension plan where over 40% of an individual account's value is consumed by various charges and fees. They provide possible explanations for their findings but also raise several caveats: For instance, their results are quite sensitive to mortality assumptions, and they are focussing on the mean of the money's worth of an annuity without considering its distribution.

Annuitization costs, i.e. the positive difference between *actuarially fair* and market-observed annuity prices, arise endogenously in the model presented in a paper by Van de Ven and Weale (2006). In their two-period overlapping generation model, they consider the effect of aggregate longevity uncertainty on annuity rates. Within an equilibrium framework they show that given risk-averse preferences, annuities are sold at a discount for aggregate mortality risk. The value of this discount is very sensitive to both, the changes in the risk aversion parameter and the level of uncertainty regarding the future lifetime.

All-in-all there tends to be an accordance among empirical and theoretical approaches that annuities are not fairly priced, i.e. they include a "transaction cost" which at least partially consists of contingency funds and can thus be interpreted as a risk discount. However, the question of whether this risk discount is adequate regarding the inherent risk is open, and it is not clear whether it is majorly influenced by investment or longevity risk, that is the risk that future mortality patterns differ from those anticipated in a way that is unfavorable from an annuity provider's point of view.

While most of the above cited articles regard annuities from a pensioner's point of view, we

[1] In general, mortality rates for the overall population exceed those for individual annuity purchasers when comparing them within the same year and cohort.

take a different approach by considering the annuity business from the provider's position. From an insurer's perspective, selling annuities is similar to assuming a short position in a forward contract on the survival of the annuitants within the insurer's portfolio. But in contrast to standard forward contracts, this position cannot be hedged as, thus far, there are no appropriate securities.[2] However, similar to classical portfolio selection problems, the risk-return profile of the insurer's position can be analyzed.

So far, there have been few contributions assessing the risk within life contingencies, and according to Dowd et al. (2006) insurance companies have a general problem assessing the magnitude of the financial risk implied by their mortality exposure. Using the two-factor stochastic mortality model from Cairns et al. (2006b), Dowd et al. (2006) give a "blueprint" how to estimate risk measures for any mortality-dependent risk and provide four illustrative types of mortality dependent financial positions, including an insurer's annuity book hedged by a hypothetical longevity bond similar to the one announced (but never issued) by the European Investment Bank and BNP Paribas in November 2004 . However, their analysis is rather geared towards the general study of mortality contingent positions and does not provide a survey of the risks within an annuity book.

The present study aims at filling this gap: We present a simple long-term analysis of risks of an annuity book and, in particular, provide qualitative insights on how influential mortality risks and investment risks are, respectively. Furthermore, the implications of this preliminary analysis are discussed. More specifically, we focus on the question of whether annuities are priced appropriately regarding the inherent risk.

In order to assess the risks within annuities, forecast models are needed for both, the evolution of mortality and investment risk. The latter, i.e. modeling the evolution of financial assets, is a well-studied problem with many available models. For forecasting the evolution of mortality, on the other hand, the list of appropriate models is relatively short. For a detailed overview and a comprehensive categorization of stochastic mortality models, we refer to Cairns et al. (2006a). One of the most prominent models is the so-called Lee-Carter model (see Lee and Carter (1992)) with its various extensions (see, for example, Lee (2000), Brouhns et al. (2002), or Renshaw and Haberman (2003)). This approach quickly gained acceptance in both the academic community and among practitioners.[3] Due to its popularity and relative simplicity, we choose a variant of the Lee-Carter method for forecasting mortality and a very simple approach for modeling the evolution of the financial market.

The remainder of the text is organized as follows: In Section 2, we introduce the annuity book and the forecasting approaches. Section 3 describes our simulation framework as well as the underlying data. Subsequently, our results are presented in Section 4, with a discussion in Section 5. Finally, in Section 6, we summarize, discuss the limitations of our results, and provide an outlook for future research.

2 Modeling the Risks within an Annuity Book

According to Dowd et al. (2006) in order to assess the magnitude of the financial risk implied by an insurer's mortality exposure the key ingredients are a stochastic model of aggregate mortality, a selection of financial risk measures, and a simulation framework. In order to capture the dimension of the total financial risk implied by an insurer's annuity book we further need a stochastic model for the insurer's assets.

In this section, after introducing the contract and the "risk" to be studied in Subsection 2.1, we present our choices of asset and mortality models in Subsections 2.2 and 2.3, respectively.

2.1 The Annuity Book

An annuity is an insurance contract providing payments for the remaining lifetime of the insured person, i.e. it is contingent on the insured's survival. Annuities are offered in various designs, such as

[2] An exception are so-called longevity bonds or other longevity derivatives, i.e. financial instruments with payoffs contingent on the survival of a certain cohort or population, which are starting to be offered by several investment banks (see, e.g., Blake et al. (2006)).

[3] For example, it served as a starting point for mortality forecasting models used by the U.S. Social Security Administration.

deferred or immediate annuities, variable or fixed, with or without guarantee periods, etc. We focus on the simplest kind, namely a fixed single premium immediate annuity (SPIA) without any guarantee period, and this is also one of the predominant types of contract especially in the UK market. The annuity market can be divided into open market options and compulsory purchase annuities. While according to Finkelstein and Poterba (2002), the compulsory market is considerably larger than the voluntary market, we focus on prices for the open market option. Thus, the structure of the considered contract is quite simple: At time zero the annuitant pays a certain single premium, say P, to the annuity provider. In return (neglecting credit risk) the annuitant obtains a payment of U units every period conditional on her or his survival.

Therefore, assuming an initial expense of ϕ units, a policyholder's individual account value A at time 1 in the case of survival is

$$A_1 = (P - \phi)(1 + \delta_1) - U\, 1_{\{\tau > 1\}},$$

where δ_t is the charge-adjusted return on the asset process governing the policy reserve fund from time $(t-1)$ to time t, and τ is the time of death of the recipient. Similarly, the account value at time t – given the insured's survival until $(t-1)$ – is given by

$$A_t = A_{t-1}(1 + \delta_t) - U\, 1_{\{\tau > t\}}. \tag{1}$$

Assuming that the insurer's portfolio of policies is large enough to neglect unsystematic mortality risk[4] and considering x_0-year old males at time 0, equation (1) can be transformed to express the reserve per unit annuity (i.e. $U = 1$) at time t as

$$R_t = R_{t-1}(1 + \delta_t) - {}_t p_{x_0}, \tag{2}$$

where ${}_t p_{x_0}$ denotes the (realized) proportion of the population of x_0 years olds at time 0 who are still alive at time t, i.e. who survived t periods from time 0 to time t. It is worth noting that R_t is not known prior to time t and thus is a random variable as both δ_t and ${}_t p_{x_0}$ are random variables that are $\mathcal{F}_t$-measurable.[5]

If ω denotes the limiting age, i.e. if survival beyond age ω is not possible, there are no payments after time $T = (\omega - x_0)$. The remaining surplus R_T then characterizes the profitability regarding this particular book of annuities from the insurer's point of view:

- If $R_T < 0$, the reserves have not been sufficient to cover the insurer's liabilities. The resulting shortfall has to be financed using other funds.
- If $R_T \geq 0$, there have been enough reserves to settle all claims, and the remaining surplus remains with the insurer as a profit.

Therefore, in order to assess the risk of a book of immediate annuities, it is sufficient to analyze the properties of the random variable R_T, which depends on the evolution of both, the insurer's assets and liabilities.

2.2 Modeling the Insurer's Assets

We assume that at time zero, the insurer invests collected premiums in three different types of assets: zero-coupon bonds, a savings account B, and a well-diversified stock portfolio S. We further assume that non-defaultable zero coupon bonds for any maturity t exist and that they are available for a price of $p(0, t)$ at time 0.[6] Thus, the amount invested in a bond with maturity t accrues at a constant rate of return rate over the period $[0, t]$, while returns on funds invested in B and S yield a stochastic return corresponding to the evolution of the interest rate and the stock portfolio, respectively. Hence,

[4]Here, *unsystematic mortality risk* corresponds to the part of the risk that can be diversified by the Law of Large Numbers. See Biffis et al. (2005) for a classification of risks affecting insurance securities.

[5]As usual in this context, we fix a complete filtered probability space $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F})_t, P)$ for our consideration, where the filtration $\mathbf{F}$ describes the information flow.

[6]The assumption that bonds are available for all maturities and in particular that ultra-long bonds exist may seem unrealistic. However, some governments have recently begun or are planning to issue bonds with maturities of 30 years or even beyond.

we need to make assumptions about the dynamics of B and S. While numerous models with different levels of sophistication are available we rely on simple yet common specifications.

The savings account B evolves according to the differential equation

$$dB_t = r_t\, dt,$$

where $r = \{r_t\}_{t>0}$ is the nominal short-rate of interest, and it is modeled by a mean-reverting square-root process

$$dr_t = \psi(\gamma - r_t)\, dt + \sigma_r \sqrt{r_t}\, dW_t, \tag{3}$$

i.e. we assume the well-known Cox-Ingersoll-Ross (CIR) model for the evolution of the short rate (see Cox et al. (1985) for details). Here, ψ, γ, and σ_r are some positive constants and W is a one-dimensional standard Brownian motion. Furthermore, we model the stock portfolio S by a geometric Brownian motion, i.e. we let

$$dS_t = S_t\left(\mu\, dt + \sigma_S\, dZ_t\right), \tag{4}$$

where again μ and σ_S are some positive constants and Z again is a one-dimensional standard Wiener process, assumed to be independent of W. In particular we assume independence of the asset and the short-rate process.

Having specified the underlying processes, we need to assume certain allocation strategies in order to obtain the evolution of the returns on the reserve δ from equation (2). We consider two strategies:

1. The insurer does not hedge the liabilities and invests the entire funds into the savings account and the stock portfolio at fixed proportions $(1-\alpha)$ and α, respectively, and we also assume that the portfolio is continuously rebalanced. Thus, α describes the stock proportion within the insurer's asset portfolio.

2. The insurer attempts to hedge the liabilities by buying ${}_t\hat{p}_{x_0}$ bonds with maturity t for $t = 1, \cdots, T$, where ${}_t\hat{p}_{x_0}$ denotes the best estimate of his (random) liabilities ${}_tp_{x_0}$ at time t. The remainder of the reserve is then invested into the same portfolio as in strategy 1. If the payoff from a t-bond exceeds the liabilities, the excess amount will also be transferred to this portfolio. Similarly, if the bond payoff is insufficient, funds from the portfolio are used to settle the insurer's obligations. Thus, it is sufficient to simulate the short-rate and the stock portfolio as bond prices are only needed at time 0 for which market prices are available. In this case δ refers only to the excess or loss amount invested into or borrowed against the variable portfolio.

These two asset strategies are both somewhat extreme: In the first *opportunistic* strategy, the insurer does not try to match his assets to his liabilities at all, whereas in the second case assets are matched to liabilities as well as possible at time zero. However, the considered strategies do not allow for adjustments based on the information at time t but are $\mathcal{F}_0$ measurable. Thus, when continuously rebalancing the portfolio according to the information available, the insurer could even improve the hedge in strategy 2.

Even though actual investment strategies will be somewhere in between, we believe that these two scenarios will provide adequate insights on the *actual* distribution of R_T. However, there are some caveats: When funds are not sufficient to cover the liabilities, i.e. if the reserve R_τ becomes negative for some $0 < \tau < T$, these strategies imply that the insurer will finance the shortfall by borrowing against a variable portfolio rather than borrowing money at the prevailing interest rate. While this assumption seems unrealistic at a first glance a large annuity provider might be able to "borrow" funds from some other separately accounted portfolio or line of business within the company or its group. Nevertheless, negative final reserve accounts have to be treated with care with respect to the actual amount of the shortfall, but *not* the fact that they are negative.

2.3 Modeling the Insurer's Liabilities

According to equation (2), it is sufficient to model ${}_tp_{x_0}$ appropriately. Recently, it has become clear that mortality improvements behave in an unpredictable manner (for an assessment of future mortality trends, see, e.g., Currie et al. (2004)). Therefore, it is not sufficient to use estimates from prevailing mortality tables but their stochasticity has also to be taken into account.

So far, several stochastic mortality models have been proposed; for a detailed overview and a categorization see Cairns et al. (2006a). One of the most prominent stochastic models for mortality was proposed by Lee and Carter (1992, henceforth LC). This approach has rapidly gained acceptance not only in academia but also among practitioners, which may partly be due to its relative simplicity.

In the LC model logarithmic central death rates $m_{x,t}$ for a life aged x at discrete times t, or during years $[t, t+1)$, are assumed to follow the equation

$$\ln m_{x,t} = a_x + b_x \kappa_t + \varepsilon_{x,t} \tag{5}$$

where a_x and b_x are age-dependent parameters, κ_t is a so-called time index, which is independent of age, and $\varepsilon_{x,t}$ is the error term with $\varepsilon_{x,t} \sim N(0, \sigma_\varepsilon^2)$. We impose the restrictions $\sum_t \hat{\kappa}_t = 0$ and $\sum_x \hat{\beta}_x = 1$ which ensure the uniqueness of the representation.

However, the LC model in its original form has a number of shortcomings, one of which is the assumption that error terms are homoscedastic, i.e. $\varepsilon_{x,t} \sim N(0, \sigma_\varepsilon^2) \quad \forall x, t$. Alho (2000) – among others – pointed out that this assumption is somewhat unrealistic. They argue that in a number of cases residuals actually showed significant variation with respect to the age x.

Among a large number of extensions and propositions for improvement is an approach presented by Brouhns et al. (2002). They resort to the notion that the number of deaths among people aged x in a given time interval $[t, t+1)$, $d_{x,t}$, can be regarded as a Poisson counting process, which goes back to Brillinger (1986). The assumption, however, implies that error terms are heteroscedastic, i.e. $\text{Var}(\varepsilon_{x,t})$ may vary by age x, which seems to be more realistic. Another advantage is the fact that parameters (which have virtually the same interpretation as in the LC model) can be obtained iteratively as maximum likelihood estimators using an algorithm developed by Goodman (1979). Due to the more realistic assumptions we resort to this alternative approach for our further analysis of mortality. Henceforth it will be denoted by PML (*Poisson Maximum Likelihood*).

To project the evolution of mortality, only the time index $\hat{\kappa}_t$ needs to be projected since $\hat{a}_x$ and $\hat{b}_x$ depend on the age x only – but not of time t. Following the standard time series methodology of Box-Jenkins, we choose an ARIMA(0,1,0) process, i.e. a random walk with drift, for $\{\hat{\kappa}_t\}_{t>0}$:[7]

$$\hat{\kappa}_t = \hat{\kappa}_{t-1} + \theta + \eta_t. \tag{6}$$

By projecting the time series $\{\hat{\kappa}_t\}_{t>0}$, mortality rates for a person aged x_0 in year 0 can be forecasted for future years $s = 1, 2, \ldots$ through

$$\ln \hat{m}_{x_0+s,s} = \hat{a}_{x_0+s} + \hat{b}_{x_0+s} \hat{\kappa}_s. \tag{7}$$

Central death rates $m_{x,t}$ are widely used by actuaries and even more so by demographers to describe and explain mortality. However, they are less illustrative than death or survival probabilities. Following the standard notation (see e.g. Bowers et al., 1997) we transform the central death rate projections from (7) into survival functions. For a person aged x_0 at time 0, the survival function $s_{x_0}(s) = {}_s p_{x_0}$ gives the s-year survival probability ($s > 0$), the latter being the notation widely used by actuaries. Restricting life time to a limiting age of ω by setting $p_{\omega-1} = 0$ it can be expressed as

$$\begin{aligned} {}_s p_{x_0} &= \prod_{j=0}^{s-1} p_{x_0+j} = \prod_{j=0}^{s-1} \exp(-\mu_{x_0+j}) = \prod_{j=0}^{s-1} \exp(-m_{x_0+j,j}) \\ &= \prod_{j=0}^{s-1} \exp(-e^{\hat{a}_{x_0+j} + \hat{b}_{x_0+j} \cdot \hat{\kappa}_j}). \end{aligned} \tag{8}$$

For the transformation in (8), we use the frequently made assumption of constant force of mortality in each year, i.e. $\mu_{x,t+u} = \mu_{x,t} \quad \forall u \in [0, 1)$ (see, e.g., Bowers et al. (1997) for further details).

[7]Although any ARIMA(p,d,q) process could have been used, the ARIMA(0,1,0) model has previously been favored by numerous authors for varying data sets.

The drift parameter (or trend) of the ARIMA(0,1,0) process from (6), θ, is estimated from the fitted values of $\hat{\kappa}_t$. The disturbance terms $\eta_t \sim N(0, \sigma_\kappa^2)$, the variance $\sigma_\kappa^2$of which can also be estimated directly from fitted values of $\hat{\kappa}_t$, solely incorporate randomness. By transforming (6) we directly obtain randomized forecasts of the time-index when adding randomized values of η_t. Thus, by combining (8) and (6) random realizations of the survival function $s_{x_0}(s)$ are generated.

While the presented PML approach extends the well-known LC model and overcomes some of the shortcomings of the original model, an important caveat must be made. As is the case with all attempts to model mortality, we do not know whether mortality actually behaves in the particular way assumed by the LC and/or the PML approach. This uncertainty, i.e. the question of whether a chosen model is the correct model or not, is commonly referred to as model uncertainty. Apart from this uncertainty, we also have to be aware of a number of shortcomings of the LC model that could not be corrected by the PML approach. To mention only some of these potential weaknesses, the LC model has been shown not to be able to sufficiently incorporate cohort effects[8]; furthermore the LC model has been criticized for underestimating mortality risk in some situations.

After having specified our asset and liability modeling approaches, and recurring to the blueprint of Dowd et al. (2006), we still need to provide a simulation framework and some risk measures. These ingredients will be presented next.

3 The Simulation Framework

In the previous section we described the underlying asset and liability model for an annuity provider. However, these models still need to be calibrated. In 3.1 we describe the data. Subsequently, in order to simulate assets and liabilities simultaneously, we embed the underlying models in a simulation framework in 3.2. As our objective is to analyze the risks within annuity contracts we introduce the considered risk measures in 3.3.

3.1 Data

The evolution of the insurer's assets and liabilities depends on various quantities: annuity prices, asset yields, survival probabilities, etc. In the following, we describe the data and the procedures used to derive these quantities. Since the annuities market in the UK is one of the largest in the world we focus our analysis on data from the UK. In other annuity markets, e.g. in Germany, contracts are usually sold as participating contracts where profit distribution forms a significant component and would have to be accounted for separately.

Annuity Price

The Financial Services Authority (FSA), the regulatory body for insurance companies in the UK, regularly issues information concerning recent market prices for different types of annuity contracts in the comparative tables section on their web page. As mentioned earlier in the text we consider prices for the open market options type of annuities, payable monthly in arrears and without guarantee period. Since all mortality data was available until 2003 only, we choose January 1, 2004 as the start date for our projections. Hence we use an annuity quote for January 2004. Among the five lowest quotes, i.e. those providing the highest yearly (or monthly) payments for a fixed single premium, we chose the median provider to eliminate possible temporary effects.

Bond Prices

As our allocation strategies only include bond investments at time 0, we only need to infer bond prices at the inception date, i.e. we can use historical data. We use UK government bonds[9], where zero coupon yields are available for short-term, 5-year, 10-year, and 20-year bonds. For maturities

[8]For a detailed analysis of *cohort effects* in the UK, i.e. the effect that some cohorts participate more than others from lower mortality, cf. for instance Willets (2004). For an attempt to measure mortality improvements and compare cohort effects for a selection of countries see MacMinn et al. (2005).

[9]Source: http://www.statistics.gov.uk/statbase, 10/18/2006.

in between we interpolate linearly. As we also need bonds for longer maturities we extrapolate the available data. Here we took a conservative approach: if the yield curve was upward shaped, i.e. if the yields for bonds maturing in 20 years was higher than for bonds maturing in 10 years, we assumed that yields for longer maturities equal that for a 20-year bond, i.e. we extrapolate a flat yield curve. If the curve was downward shaped, we extrapolated linearly to incorporate sufficient prudence.

Short rates and Stocks

We use the 3-months LIBOR[10] from January 1978 to September 2006 (monthly data) for calibrating the short-rate model.[11] For determining the parameters of the stock model, we used the monthly time series of the FTSE 100 price index (January 1984 to September 2006) and of FTSE 100 dividend yields (September 2003 to September 2006).[12], and we rely on standard maximum likelihood estimators.

stock model	μ	σ_S		
	12.07%	16.20%		
short-rate model	ψ	γ	σ_r	r_0
	5.54%	8.04%	5.2%	3.99%

Table 1: Parameters of the asset models

In Table 1, the resulting parameter estimates are displayed. For the stock quota within the mixed portfolio, we use 37% as is displayed in the *Legal and General* PPFM Data Annex from 06/20/2006 for non-participating pension policies and adaptable pension plans. It is worth noting that within the second asset allocation strategy, where the majority of funds is invested in a bond portfolio, the proportion of stock within the insurers asset portfolio will be smaller, as only 37% of the remaining funds are invested in stock.

Mortality Data

We consider three different sets of mortality data: on the one hand we aim at capturing typical mortality patterns of the UK population, on the other hand typical mortality of insured persons was of interest within the scope of the present investigation. Due to selection effects, mortality rates for the general population usually exceed those for the population of assured.[13]

We resort to data from the Human Mortality Database (2006, henceforth HMD) for the overall population mortality. Since data for the entire UK was not available, we follow prior investigations and consider the series for male persons in England & Wales. More specifically, annual numbers of death and exposure to risk for ages 60-100 during calendar years 1983 through 2003 were used within the calibration procedure.

For the mortality experience of the population of insured, suitable data was cordially provided by the Continuous Mortality Investigation Bureau (CMI), and we focus on observations from years 1983 to 2003 at ages between 60 and 100 in order to ensure compliance with the HMD data. Apart from the usual per-capita data, i.e. exposures and deaths all having the same weight of 1, we also considered mortality by amounts. This accounts for varying face values of annuity contracts, and it could possibly contribute to refine results with respect to (anti-)selection effects. Note that not only do insured persons exhibit lower mortality than the general population, but there is also evidence that persons with higher income – thus being able to buy annuity contracts with higher benefits (and higher premiums) – tend to live longer than less well-off persons.

As all data sets under consideration provide only sparse data for ages beyond 100, this data is not included in our estimation procedure. Instead, we assume that if an insured person survives until just before his 101^{st} birthday, the insurer will pay an unconditional compensation – regardless of

[10]London Interbank Offer Rate.

[11]Source: http://www.statistics.gov.uk/statbase, 10/18/2006.

[12]Source: http://www.statistics.gov.uk/statbase, 10/18/2006; due to data availability we used the shorter time series for the dividend yield.

[13]See e.g. Finkelstein and Poterba (2002).

the remaining life time of that individual. This can be regarded as a means to clear the portfolio of annuities and thus to dispose of the inherent uncertainty.

Note that the expected remaining (curtate) life time[14] beyond age 100, $\mathring{e}_{101}$, gives the expected number of future annuity payments: exactly those payments that will be paid for under the notion of a lump-sum compensation. To make the value of this compensation more prudent, no discounting was included so that for a face value (i.e. yearly payments) of 1 it simply amounts to $\mathring{e}_{101}$.

For cohorts born in $z = 1983 - 101 = 1882$ through '$2003 - 110 = 1893$[15], i.e. for those for which our data sets cover *all* ages beyond 100, we determined $\mathring{e}_{101}^{(z)}$. Since there was no significant trend across these cohorts, we took the average of observed cohorts' remaining life expectancy, $\mathring{e}_{101}^{(z)}$, to estimate the compensation. For each random path of mortality, given survival until age 101, this compensation was put up to close out the particular contract at that time, i.e. at time $t = z + 101$. For further reference, note that for our investigation we fix the face value of the annuity contracts to $U = 1'$ and proceed with a contracting age $x_0 = 60$ and "limiting" age $\omega = 100$ due to the afore-mentioned compensation. Hence we have a maximum time horizon of $T = 40$ throughout our analysis.

Fees and Expenses

Annuities are subject to several fees and expenses. For a typical contract of a large annuity provider, an initial expense of 200£ plus 1.3% of the premium paid, an investment fee of 7 basis points (bps), and an additional fixed annual renewing fee are representative. However, as the influence of the fixed fees, i.e. the 200£ initial expense and the annual renewing fee, depend on the annuity purchase price, we do not consider them for our analysis. Furthermore, we assume that there are no investment fees for the bond portfolio. Thus, we only consider an initial expense of 1.3% and an investment fee of 7 bps for the risky portfolio.

3.2 Combining Asset and Liability Simulations

For both the mortality model and the development of the assets 20,000 random paths have been generated. This was done in the following fashion:

- Parameters for both models were estimated from the underlying data sets.
- Random paths for the stock portfolio and the savings account were generated using the models from Section 2.2 using the previously estimated parameters.
- Estimated parameters from the PML approach were combined with the technique described in Equation (6) to obtain randomized survival probabilities.
- Asset and liability paths were combined to obtain 20,000 randomized developments of the annuity provider's portfolio.

This process was repeated in order to account for the different assumptions made: the two investment strategies presented in Subsection 2.2 and the different data sets of mortality.

3.3 Measuring Risks

Following the approach described in Subsection 2.1, R_t, the reserve per unit annuity at time t is employed for describing the financial situation of the annuity provider. Aggregating the remaining surplus values R_{40} from all realizations of the models underlying the insurer's assets and liabilities allows us to compute the expected value $\mathbb{E}(R_{40})$, the shortfall probability $\mathbb{P}(R_{40} < 0)$, the value at risk (VaR) at probability level α, i.e. $z_\alpha^{(40)}$ such that $\mathbb{P}(R_{40} < z_\alpha^{(40)}) \geq \alpha$, and the conditional tail expectation $\mathbb{E}(R_{40} | R_{40} < z_\alpha^{(40)})$, assuming $\alpha = 0.001$ in both cases. Furthermore, by plotting histograms of the realizations, we can illustrate empirical density functions.

[14]The *curtate* life expectancy gives the (integer) number of whole years a given individual will continue to live.

[15]Note that we interpret $\omega = 110$ although the last class of ages in the HMD data denotes ages 110 and beyond. Due to very scarce data at this "age" we neglect possible inaccuracy.

The defined risk measures relate to *all* random paths generated. Hence, they reflect random coincidences of a mortality development and an asset development. "Good" and "bad" developments – with respect to either part – can potentially level out. Insurers welcome this diversification effect of mortality and interest risk. However, this phenomenon might dilute actual characteristics of either risk component.

In order to measure the magnitude of these two aspects of risk, we condition the sample described above on "good" and "bad" paths of mortality as well as of asset development. For the determination of "unidirectional conditional" risk measures, i.e. the risk measures described above restricted to "good" or"bad" random paths (with respect to either mortality *or* funds), we compose a smaller sub-sample of 2,000 of the "best" and "worst" developments with respect to one component and a random selection of developments of the other component. For example, for the "conditional" risk measures with respect to mortality, we combined the 2,000 "best" mortality paths with 2,000 randomly chosen funds paths. The same was done with the 2,000 "worst" mortality paths. On the basis of these 2,000 reserve values R_t the introduced risk measures were calculated.

In this context "good" and "bad" random paths are understood from the insurers perspective, i.e. "good" mortality development basically translates into early death since in that case the obligation to pay terminates. "Good" developments with respect to funds refer to a high average or aggregate rate of return over time since higher yields are to be seen positive. "Bad" developments refer to the respective opposite outcomes.

To make exact comparisons between any two paths, we considered the 40-year life expectancy, $\mathring{e}_{60:\overline{40}|}$, for mortality paths and the aggregate interest rate development, $\sum_{k=1}^{40} f_{0,k}$ with $f_{0,k} = \prod_{t=1}^{k}(1+r_t)$, for the funds paths. While the former gives the expected number of years that a 60-year person will continue to live within the next 40 years, the sum in the latter expression can be regarded as an approximation of the integral of the yields' evolution curve. Note that the exact values of these "measures" are irrelevant for our analysis, our comparisons are only based on the relative size of the numbers which we believe do adequately reflect the typical annuity providers' preferences in terms of (for his business) desirable mortality and interest rate realizations. Applying these "measures" to the respective mortality paths permitted to compare and rank the random developments, and thus compute the "conditional" versions of the above risk measures.

4 Results

4.1 Unconditional Analyses

To analyze the risks we focus on the risk measures previously defined under the various parameter settings detailed out in Subsection 3.2. Furthermore, as the expected value of the remaining surplus $\mathbb{E}[R_{40}]$ is in monetary units at time $t = 40$, we also provide the discounted expected value $p(0,40)\,\mathbb{E}[R_{40}]$ which is given in monetary units at time $t = 0$. However, as the return on a 40-year bond in 2004 (4.24% p.a.) is relatively low in comparison to the long term mean of interest rates, these quantities have to be interpreted with care.[16] Note that the amount invested in bonds within strategy 2 coincides with the *actuarially fair* value or the "money's worth" of the annuity product as defined by Mitchell et al. (1999) and Finkelstein and Poterba (2002). For the UK during the late 1990s the latter find that for an annuitant, i.e. taking into account selection effects, the money's worth of an annuity amounts to between 86 and 95 pence per pound annuity. For pensioners we find a value of 13.49 percent, i.e. the money's worth amounts to 0.9026 per unit annuity, which is at the upper end of the spectrum from Finkelstein and Poterba (2002). For the general population, we find a value of 11.92 percent (yielding a money's worth ratio of 0.7976) which is considerably lower due to the mentioned selection effect.

Similarly, for the remaining surplus values after 40 years, R_{40}, we can identify a significant difference between considering the overall population or restricting our analysis to insured persons' mortality. The respective values for strategy 2 are given in the second and fourth columns of Table 2; Figure 1 shows the corresponding histograms.

[16] For example, the mean reversion level of the CIR-process from equation (3) can serve as a proxy for the long term mean, and with 8.04% it is substantially higher than 4.24%.

	Pop. Lives		**Ann. Lives**	
Risk Measure	Strategy 2	Strategy 1	Strategy 2	Strategy 1
$\mathbb{E}\,[R_{40}]$	98.45	201.53	47.35	176.39
$p(0,40)\,\mathbb{E}\,[R_{40}]$	17.92	36.69	8.62	32.12
$\mathbb{P}\,(R_{40} < 0)$	0 ‰	1.1‰	0‰	7.6‰
$V@R(0.001)$	14.36	-9.45	6.81	-22.41
$\mathbb{E}\,[R_{40} \mid R_{40} < V@R]$	12.93	-11.39	6.23	-36.64

Table 2: Risk measures for R_{40}, Population Lives and Annuitants Lives

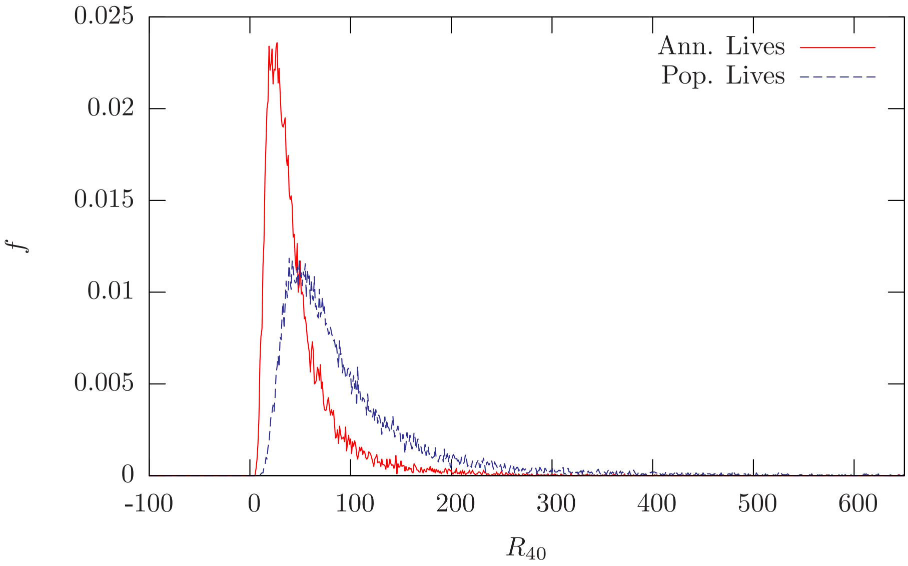

Figure 1: Histograms of R_{40} under Bond Strategy 2

Under neither parameter constellation does R_{40} become negative when applying strategy 2. Nevertheless, the selection effect of assuming insured persons' mortality is obvious. The empirical distribution in this case is somewhat closer to zero and it is less scattered than the overall population's surplus distribution. The expected value in the case of annuitants' mortality is roughly reduced by 50%. The observation for the value at risk and the conditional tail expectation are similar which translates into the insurer operating under greater exposure to longevity risk, although we should underline the fact that at least the risk of shortfall, i.e. $R_{40} < 0$, can be hedged under the assumption of strategy 2.

The pronounced selection effect in the case of bond hedging can also be observed if the insurer does not try to hedge assets and liabilities, but instead pursues an "opportunistic" investment. In that case, however, shortfalls do occur, i.e. in some cases we observe $R_{40} < 0$. As before, the insurers' surplus situation is somewhat tighter when resorting to annuitants' mortality while it is also less volatile compared to the general population case. Values of the corresponding risk measures are given in columns 3 and 5 of Table 2.

Figure 2 shows the simulated distributions of final reserves R_{40} under both investment strategies when general population mortality is assumed. As pointed out before no shortfalls occur under the bond-hedging strategy. While the expected final reserve under the opportunistic investment strategy is roughly twice what it is under strategy 2 these potential insufficiencies produce a shortfall probability of 1.1‰. Also, both the V@R at 0.001 and the corresponding Conditional Tail Expectation become negative in the case of opportunistic investment while they remain positive under bond hedging. This shows that the results are considerably more volatile if no bond hedging is available,

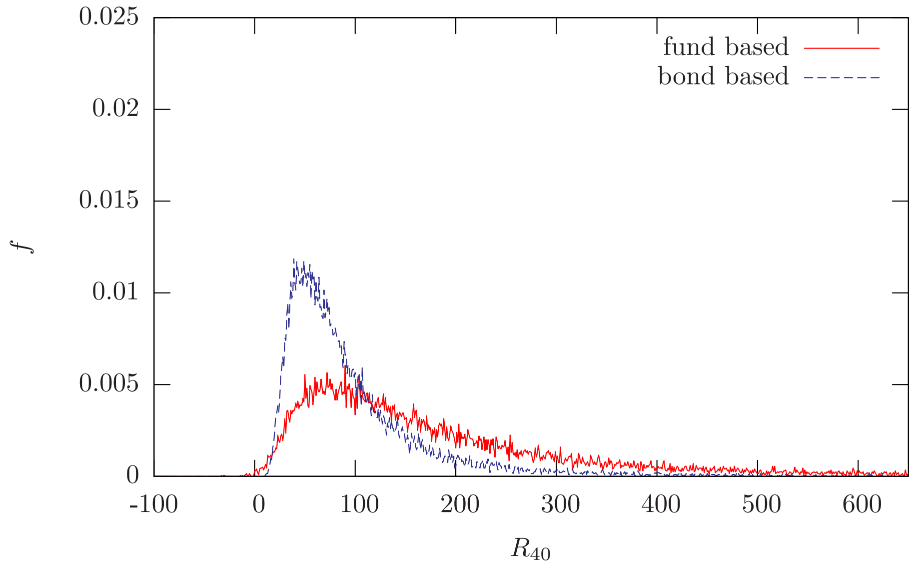

Figure 2: Histograms Population Lives

rendering the final reserve situation less predictable.

The same tendencies can be observed when assessing the results in the situation of annuitants' mortality either with respect to lives or amounts, although the specific numbers differ from the case of the general population mortality. However, as a general result for our model we should emphasize that bond hedging prevents shortfalls regardless of the specific mortality data. Contrarily, under opportunistic investment $R_{40} < 0$ does occur in all three cases, with probabilities ranging from 1.1‰ in the case of overall population mortality to 7.6‰ or 7.7‰ when incorporating the selection effects captured in annuitants' mortality data. The values are given in Table 3.

As pointed out earlier, there is a selection effect not only when considering insured persons versus the overall population, but one can also assume that people with higher "amounts", i.e. higher annuity payments, have been subject to even more prudent selection by the insurer and through self-selection. However, this selection effect is less pronounced than the one described before; the respective risk measure can also be found in Table 3.

	Ann. Lives		**Ann. Am'ts**	
Risk Measure	Strategy 2	Strategy 1	Strategy 2	Strategy 1
$\mathbb{E}\left[R_{40}\right]$	47.35	176.39	47.10	176.38
$p(0,40)\,\mathbb{E}\left[R_{40}\right]$	8.62	32.12	8.58	32.11
$\mathbb{P}\left(R_{40} < 0\right)$	0 ‰	7.6‰	0 ‰	7.7‰
$V@R(0.001)$	6.81	-22.41	6.78	-20.68
$\mathbb{E}\left[R_{40} \mid R_{40} < V@R\right]$	6.23	-36.64	6.26	-36.35

Table 3: Risk measures for R_{40}, Annuitants' Lives and Annuitants' Amounts

While under the opportunistic investment strategy the results from the population mortality and the annuitants' mortality settings differ only slightly, which can been seen by comparing Figures 2 and 3, greater differences can be observed in the case of the bond hedging strategy. For annuitants' mortality reserve values become less scattered, i.e. results closer to the expected final reserve $\mathbb{E}(R_{40})$ occur more frequently than under the overall population mortality assumption. Even though no shortfalls are generated under these two settings, the corresponding histograms show significant deviations.

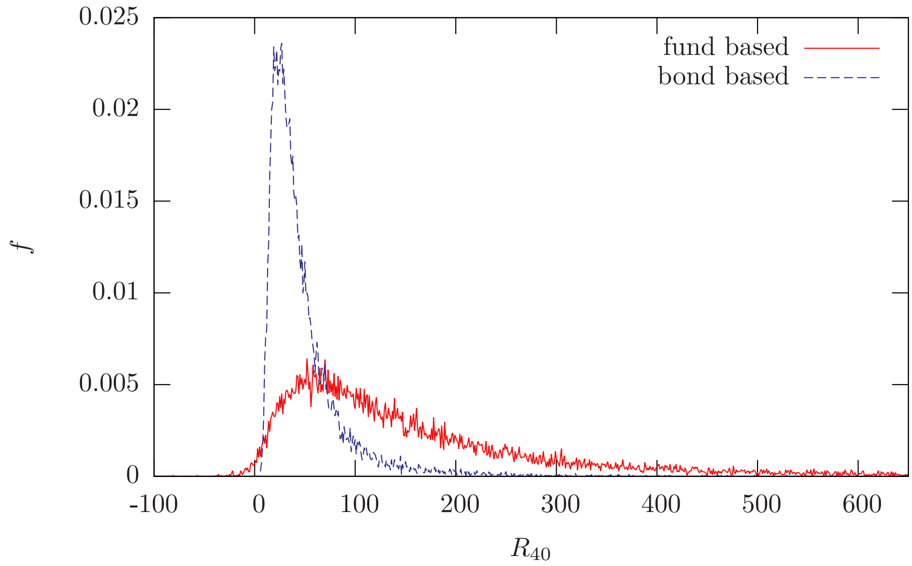

Figure 3: Histograms Annuitants Lives

To summarize the main results of the unconditional analysis, we find that the bond hedging strategy 2 allows the insurer to completely dispel the risk of generating negative final reserves. Even if the opportunistic investment strategy is pursued, which was already pointed out to be a somewhat extreme assumption, shortfalls occur relatively seldom with probabilities ranging between roughly 1‰ and 8‰.

Two selection effects could be identified. The effect of transiting from the overall population to annuitants' mortality was considerably larger than the further selection expressed by the transition from annuitants' lives to amounts which only marginally changed the results. However, the tendency is the same: the stronger the selection effect the less beneficial was the annuity provider's situation after 40 years.

4.2 Conditional Analyses

The conditional results for "good" and "bad" mortality or interest rate paths are given in Tables 4 and 5, respectively. The conditional sampling results are presented together with results for the unconditional values, which have already been discussed in the previous section, in order to facilitate a comparison of both analyses.

Risk Measure	Bad Mortality	Unconditional	Good Mortality
$\mathbb{E}\left[\tilde{R}_{40}\right]$	43.82	47.35	49.08
Rel. Deviation	-7.46%	0	+3.64%
$p(0,40)\,\mathbb{E}\left[R_{40}\right]$	7.98	8.62	8.94

Table 4: Expected values of the free reserve, conditional on mortality and unconditional, using annuitants' lives mortality under the bond-based strategy 2

As can be seen from Table 4, when restricted to "good" or "bad" mortality evolutions, these "marginal" results are relatively close to those from the unconditional analysis. Figure 4 shows the corresponding histograms, and although differences are not strikingly pronounced, the general direction is noticeable: a spread in R_{40} of almost 5 per unit means that the annuity could potentially be paid for five more years. If the difference between actual and *actuarially fair* price, i.e. the

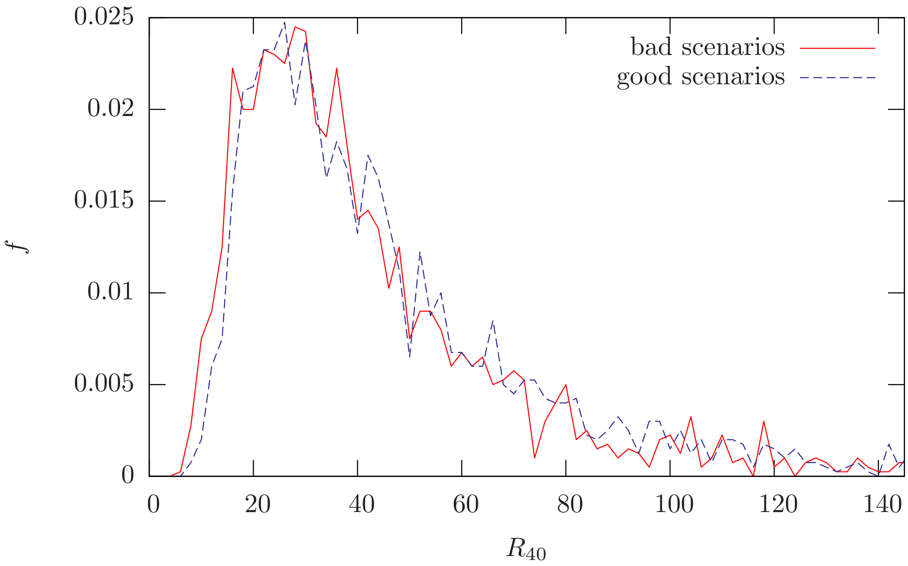

Figure 4: Comparison of influence of good and bad mortality developments

"transaction cost", was smaller the situation would be drastically different for the annuity provider. In short: there is significant risk, but due to relatively high transaction costs, the affect on the reserve situation is rather small.

Risk Measure	Lowest Asset Returns	Unconditional	Highest Asset Returns
$\mathbb{E}\left[\tilde{R}_{40}\right]$	14.16	47.35	140.45
Rel. Deviation	-70.11%	0	+196.59%
$p(0,40)\,\mathbb{E}\left[R_{40}\right]$	2.58	8.62	25.57

Table 5: Expected values of the free reserve, conditional on interest rates and unconditional, using annuitants' lives mortality under the bond-based strategy 2

In Table 5 the results conditional on "bad" and "good" yield evolutions are displayed. We observe that differences are far more pronounced in this case. This observation is underlined by Figure 5 which depicts the respective empirical distribution of R_{40}.

Moreover, we find that "bad" capital market development lead to results that are far less volatile, i.e. possible values are significantly more concentrated on a relatively small interval ranging from zero to approximately 35. This may be surprising when comparing them to the results we obtain when limiting our considerations to the 10% "best" capital market paths, in which final reserve values are generally larger but are also spread over a much larger interval. In that case the major portion of values lie somewhere in the interval between 50 and 300. Although the variation of results is somewhat opposite to what the notion of "bad" and "good" capital market paths suggests, they clearly do not lead to significant risk when thinking in terms of shortfalls, i.e. $R_{40} < 0$.

Thus, all in all, the observations are in compliance with the frequently assumed statement that mortality risk in annuities is a lot smaller than the risk arising from the capital markets. However, one should note that interest rate risk can be hedged, which is visible when comparing the bond hedging strategy with the opportunistic investment. While the latter is a situation where the insurer is completely exposed to interest rate risk, investment strategy 2 hedges that risk by investing in an appropriate bundle of bonds in advance.

Although fluctuations generated by unforeseen mortality paths tend to be considerably smaller they can still be of great interest for an annuity provider since mortality risk cannot yet be hedged

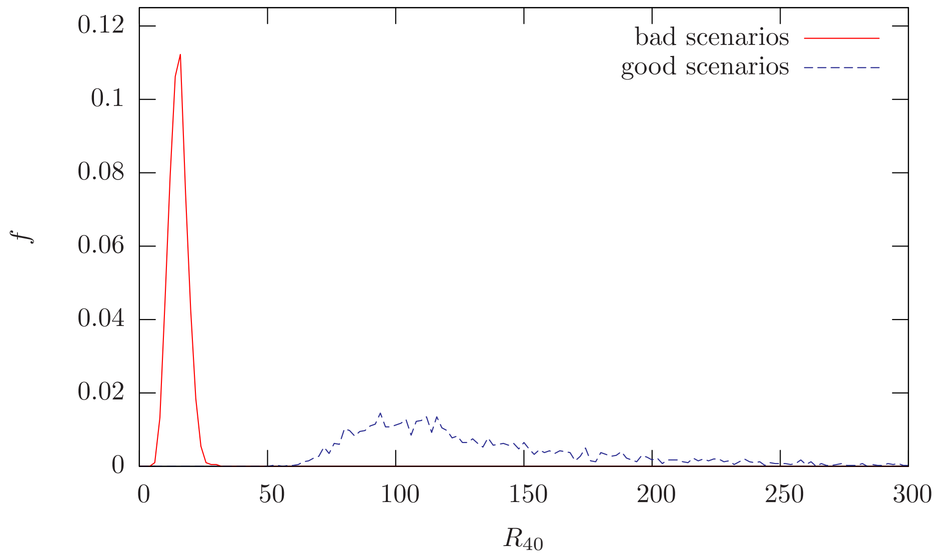

Figure 5: Comparison of influence of good and bad capital market developments

with appropriate market instruments. Hence mortality fluctuations – despite their smaller influence on the aggregate risk as outlined before – may pose a problem for annuity providers.

5 Discussion

In the last section, we analyzed the risks within a book of annuities. One of the key findings is that under almost all circumstances, the annuity provider will be able to meet the contractual obligations. Even under a very opportunistic asset allocation strategy, conditional on a (from the insurer's perspective) negative evolution of the mortality, if people tend to live extremely long, and under a very disadvantageous evolution of the financial market, the shortfall probability, i.e. the probability that the insurer cannot fulfill his obligations, is very low. This is mainly due to the considerable contingency funds.

Thus, our results indicate that – under the assumptions made – this remaining difference between actual and actuarially fair annuity price, which can be considered to describe a transaction cost, is sufficient to sustain even very disadvantageous scenarios with a profit. But can the whole costs be used to fulfill the insurer's obligations if necessary? Or, are there other cash flows that need to be included?

According to Mitchell et al. (1999) and Friedmann and Warshawsky (1990), this transaction cost is mainly due to expenses, profit margins, and contingency funds. However, Milevsky and Young (2007) point out that for variable-payment annuities there is an explicit mortality risk fee. They believe that while some economists might classify any additional fee as transaction costs, this particular fee is an inseparable component of aggregate mortality risk and creates a unique impediment to annuitization. As there must be similar fees within fixed annuities, a substantial part of the costs assessed in the present paper will also be a mortality risk fee.

Under this assumption, however, the question arises whether this fee is adequate. And here our simulation study suggests that it is not. In fact the transaction cost proves to be sufficient to cover the insurer's obligations in almost all cases, even under risky allocations where huge profits are possible (see Table 2).

Moreover, our conditional analyses show that mortality risk is far less influential than financial risk. However, the latter – at least for the most part – is hedgeable. The fact that the shape of the distribution of the remaining surplus is still more influenced by the financial risk even under this hedge, i.e. under the bond-based strategy, is also due to the high level of transaction cost: after the hedge portfolio is set up, the difference between actual and fair value is invested in the fund and therefore the performance of the insurer's asset portfolio is still highly dependent on the performance of the financial market.

As shown by Davidoff et al. (2005), full or at least partial annuitization remains optimal in many situations, even when annuities are not actuarially fair. Thus, our results are certainly not able to explain the annuity puzzle. However, they may give rise to one possible solution: an increase in the loading leads to a proportional drop in annuity payments for a given annuity premium, which, ceteris paribus, makes the annuity less attractive as an investment opportunity from the retiree's perspective. This causes a shrinking annuity demand (see Mitchell et al. (1999) or Schulze and Post (2006)), which in turn means that a decrease in the loading will lead to an increasing annuity demand. But how could the price difference be reduced?

As pointed out by Dowd et al. (2006), insurance companies seem to have a general problem with assessing the magnitude of the financial risk implied by their mortality exposure. However, with the increasing number of models and contributions of how to assess and manage aggregate mortality risk, insurers will become more familiar with this risk.[17] As a consequence of this *education*, it is likely that insurers will dare to offer cheaper annuities in the future.

One promising way to manage longevity risk is securitization via so-called mortality derivatives, for example longevity bonds, which have been the subject of lively discussions in the actuarial community in the recent past (see, e.g., Blake et al. (2006) and references therein). The basic idea of a longevity bond is that the coupon payments depend on the proportion of survivors of a certain cohort or population. Thus, if properly designed, these instruments could help annuity providers hedge most of their aggregate mortality risk while the other end of the risk could be taken by the market. And as these are investment opportunities with a rather low correlation to "regular" or traditional asset classes, the market's appetite could be considerable while the hedge for the insurers could be relatively cheap.

6 Conclusion

This paper presents an analysis of the risks within a book of single premium immediate annuities. We use historical prices of annuities from the UK and simulate the insurer's assets and liabilities simultaneously. For the simulation, stochastic models for assets and liabilities are needed; we decided to use well-known and well-studied but rather simple models, and calibrate them to UK financial and mortality data, respectively. In order to simulate the assets, we have to assume asset allocation strategies for the annuity provider. We introduce two very simple, yet extreme strategies: in the first strategy, the insurer does not try to hedge the liabilities and invests the entire reserve in a fund consisting of a risky and a locally risk-free asset (bank account) at constant proportions, whereas in the second strategy, the insurer tries to hedge liabilities as well as possible, seen from the inception date, by buying zero-coupon bonds to cover expected payoffs in each year.

By Monte-Carlo simulation, we derive the distribution of the remaining per contract surplus for an annuity paying one unit each year after the portfolio has run off, i.e. after all the annuitants within the insurers portfolio have deceased. Aside from providing plots of the density functions of the remaining surplus for the different allocation strategies and different underlying data sets, we compute several risk measures to characterize the risk-return profile of the insurer's position.

Our results indicate that the insurer's position is not very risky at all. Even under the rather opportunistic first allocation strategy, the shortfall probability, i.e. the probability that the insurer will not be able to fulfill obligations, is 1.1‰, and the expected discounted surplus is more than 36 units, i.e. more than twice the initial price of the annuity, if we calibrate the mortality model to the general population. If we use annuitants' mortality data, the situation changes tremendously: With 7.6‰, the shortfall probability is almost seven times as high, and the expected discounted surplus is decreased by more than 12% to approximately 32 units. Thus, our results are in line with earlier contributions who found pronounced selection effects within annuities.[18] However, the shortfall probability is still very low and the expected surplus rather high. Contrarily, if we consider

[17]For example, the last issue of *The Journal of Risk and Insurance* in 2006 was completely dedicated to longevity risk.

[18]See, for example, Finkelstein and Poterba (2002) or Mitchell et al. (1999).

the more conservative bond-based strategy no shortfalls occur at all. Therefore the "transaction cost" within the annuity's premium seems to be more than sufficient to cover the insurer's obligations, even under rather disadvantageous scenarios.

In order to compare the influence of the aggregate mortality risk and the financial risk within the annuity book we present a series of conditional analyses: by restricting our simulation to (from the insurer's perspective) advantageous or disadvantageous mortality and financial scenarios, respectively, we are able to assess how "bad" or "good" mortality or financial environments affect the insurer.

We find that the performance of the financial market is far more influential for the distribution of the remaining surplus than the evolution of mortality improvements. However, at least within the bond-based strategy, this is mainly due to the enormous safety loading as it is invested in the rather risky fund. This is also the key reason why the distribution of the remaining surplus for the bond-based strategy is altered considerably when switching from financially beneficial evolutions to non-beneficial evolutions and vice versa. However, for the bond-based strategy there are still no shortfalls for disadvantageous capital market evolutions as the "necessary part" of the interest rate risk is hedged by the bond portfolio. While an advantageous or a disadvantageous evolution of mortality improvements is clearly noticeable in the distribution, their respective influence is far less pronounced. The difference in the discounted expected value of the remaining surplus for "good" and "bad" scenarios amounts to less than one unit. Thus, a safety loading which is considerably smaller than the "transaction cost" found in the insurance premium would be sufficient.

Under the assumption that a considerable part of this "transaction cost" represents a premium for aggregate mortality risk, our study suggests that insurers charge too much for it. If insurers were to charge less, i.e. if they offered annuities at a lower price, the annuity demand would be stimulated, and this would at least partly alleviate the *annuity puzzle*. However, as pointed out by Brown and Orszag (2006), this only holds "*to the extent that consumer demand is responsive to pricing*" – and this is difficult to assess since no price elasticities exist for these products.

The notion that insurers charge too much could be explained by the general problem insurance companies seem to have with assessing the magnitude of the financial risk implied by their mortality exposure.[19] However, with an increasing number of contributions from the recent past regarding aggregate mortality risk, we believe that annuity providers will become more familiar with assessing and managing their longevity risk, which could lead to "more fair" annuity prices. For example, one promising way to manage longevity risk could be the introduction of so-called longevity bonds[20] or other mortality-linked securities to the market, enabling insurers to hedge most of their aggregate mortality risk at a relatively low cost.

Even though our results are rather strong, it would not be appropriate to rely on them without further research. In particular, with regard to the underlying models for the assets and liabilities as well as their parameterizations, caution is required. Another critical point is the fact that we restricted our analysis to the influence of aggregate mortality risk but ignored the influence of unsystematic mortality risk arising due to the finite number of annuitants within an insurer's portfolio. Also, there are open issues regarding costs and fees within annuity products, meaning our assumption that the "transaction cost", i.e. the positive difference between the actual and the actuarially fair price, can solely be interpreted as a premium for aggregate mortality risk is questionable.

Due to all these possible pitfalls, our findings and conclusions need to be interpreted carefully. More specifically, the quantitative outcomes need to be handled with care. However, the distinctness of our results support the hypothesis that annuities are with respect to the inherent risk. In order to further investigate this hypothesis, there is a need for more advanced empirical studies as it could have important implications for the *annuity* and other *puzzles*.

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[19]See Dowd et al. (2006).

[20]See, e.g., Blake et al. (2006).

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